Abstract

Compensation not only provides incentives to existing managers but attracts new managers to the firm. This paper examines the dual incentive and sorting effects of performance pay in a simple contracting model of endogenous participation. The main result is that sorting dampens optimal pay-performance sensitivity (PPS). This occurs because PPS beyond a nominal amount transfers unnecessary rent from the firm to the manager. The result helps explain why empirical estimates of PPS are much lower than predictions from models of moral hazard alone. Finally, the model delivers a number of comparative statics that can be tested against data, predicting a web of relationship between PPS, the quality of the manager, the variation between managers, and the manager’s risk aversion and outside option.
1 Introduction

Managerial compensation serves many functions. It provides incentives, attracts talent, ensures retention, provides feedback, and communicates the goals and objectives of the firm. And yet, the vast majority of the theoretical and empirical work on executive pay only considers its incentive effects. Executive contracts not only provide incentives to the existing manager, but also attract future managers to the firm, either from internal or external labor markets. Thus, performance pay has a sorting effect, in that it sorts the potential pool of managers tomorrow in addition to providing incentives to the incumbents today. The objective here is to understand the sorting effects of performance pay, namely, how the firm will solve the dual problem of providing incentives ex-post and sorting new managers ex-ante.

The main result is that sorting dampens optimal pay-performance sensitivity (PPS). While a small amount of performance pay is necessary to attract high quality managers to the firm, excessive performance pay transfers unnecessary rent from the firm to the manager. As such, excessive performance pay is costly to the firm, and thus sorting exerts downward pressure on PPS. Because the firm must also provide incentives to managers once they are hired, the incentive effect exerts upward pressure on PPS. The optimal PPS balances these twin competing effects. The downward pressure on PPS from sorting brings the theoretical predictions on PPS closer to empirical estimates. Existing estimates of PPS (0.325%, according to Jensen-Murphy, 1990) are much lower than predicted by the canonical model of moral hazard alone, even when factoring in risk aversion.

I adopt a simple contracting framework that permits a solution to the dual sorting and incentive problems. Managers have private information on their ability, and the firm’s contracts are incomplete, in that they cannot easily extract this private information through a complex menu of contracts.\(^1\) The firm proposes a contract, which consists of a salary and bonus, representing the fixed and variable components of compensation (in practice, this takes the form of cash and stock or stock options). Based on this contract, the manager decides whether to join the firm, and if he does, he exerts productive effort.

\(^1\)This amounts to a restriction of communication between the firm and the agent through the contracts. The firm cannot tailor its contracts to the full complexity to the manager’s private information. Baker and Jorgensen (2003); Lazear (2004); Melumad, Mookherjee, and Reichelstein (1997); and Ray (2007b) make a similar assumption. Bushman, Indjejikian, Penno (2000) also work in a world of private pre-decision information.
After nature resolves production uncertainty, the output is realized and the firm pays the manager based on the negotiated contract.

The crux of the analysis rests on the firm’s joint choice on salary and bonus, where the bonus measures the pay-performance sensitivity of the manager’s compensation. I start with a benchmark model of sorting alone (without incentives) and find that PPS beyond a nominal amount only transfers unnecessary rent from the firm to the manager. This downward pressure on PPS from the sorting effect is a robust phenomenon, and is a key component of the more general model which combines sorting and incentives. Within the model, compensation induces participation ex-ante as well as determining effort (and therefore profits) ex-post. The firm selects the salary and bonus jointly to equalize the marginal rates of substitution between these ex-ante and ex-post effects. In equilibrium, the firm trades off salary and bonus at the same rate, for the dual purpose of inducing participation (sorting) and creating profits (incentives).

Risk-aversion deepens this tension. When the manager’s risk aversion is small, salary and bonus are substitutes, just as with a risk-neutral manager. If the firm raises the bonus, this attracts more managers to the firm, so the firm must lower the bonus in order to keep participation unchanged. Thus, salary and bonus are equivalent instruments in achieving sorting. But if the manager is sufficiently risk-averse, salary and bonus become complements. Now, raising the bonus loads risk onto the manager, who requires a larger cash payment to compensate for this increased risk. In this case, the firm will adjust the salary and bonus in the same direction in order to induce participation.

My paper is closest in spirit to Dutta (2008) and Baker and Jorgensen (2003). Both operate in a LEN (linear contract, exponential utility, normal errors) framework, and consider an agent whose ability affects output; for Dutta (2008), effort and ability are substitutes ($e + \theta$), while for Baker and Jorgensen (2003), effort and ability are complements ($\theta e$), as in my model. Dutta (2008) allows communication, so the firm offers a menu of contracts to the manager; my model shares the assumption of no communication, as in Baker and Jorgensen (2003). Both papers find that the optimal pay-performance sensitivity falls in the variance on output, but may rise in the variance of the agent’s ability distribution (Dutta (2008) calls this information risk, while Baker and Jorgensen (2003) call this volatility).²

²Moreover, the assumption of communication and the fact that effort and ability are perfect substitutes allows Dutta (2008) to characterize the optimal contract, whereas Baker and Jorgensen (2003) derived comparative statics without solving for the optimal contract in closed form.
Why does this occur? Baker and Jorgensen (2003) work in a world of pre-decision information, in which the agent receives information before choosing his effort level, thus increasing the amount of information the agent has. The principal strengthens the incentive contract in order to induce the agent to respond more closely to his private information. The key assumption in Dutta (2008), on the other hand, rests on ability-contingent outside options. With communication, the principal’s screening contract must balance the dual problem of providing effort incentives and minimizing informational rents. As the variance in the agent’s ability increases, the asymmetric information rises, and rent extraction becomes more critical to the principal. When outside options are very sensitive to the agent’s ability, the principal lifts the bonus in order to reduce information risk.

My model shares more assumptions with Baker and Jorgensen (2003) but follows the approach of Dutta (2008). Like Baker and Jorgensen (2003), I work in a world of pre-decision information, assume ability and effort are complements, and disallow communication between the principal and agent. Like Dutta (2008), I am able to characterize the optimal contract (though implicitly) and routinely make comparisons with the benchmark moral hazard model without information. My primary difference from both papers is that I consider endogenous participation. Both papers assume the principal will hire even the worst type of manager, whereas I show that there are some managers that are not profitable for the firm. As such, the contract in my model must solve the dual problem of participation versus incentives.

Models of adverse selection and moral hazard each enjoy voluminous theoretical literatures (see Baiman, 1991 and Hart and Holmstrom, 1987 for surveys). But there have been only limited attempts to combine both in a single model. The fusion has proven notoriously difficult and researchers have made simplifying assumptions in order to make the analysis tractable. Jullien, Salani, and Salanie (2001) derives some preliminary results on a joint moral hazard and adverse selection model, though it is difficult to draw general conclusions from their analysis. Sung (2005) makes progress in a continuous time framework and Darrough and Stoughton (1986) examines the joint problem in the special context of financial context, while Bernardo, Cai, and Luo (2001) operates in the world of capital budgeting. Hagerty and Siegal (1988) shows that contracts under moral hazard and adverse selection are observationally equivalent. In general, combining the moral hazard and adverse selection model has proved notoriously difficult and, as such, the literature remains largely separate even though real life contracts must solve both
problems simultaneously. Armstrong, Larcker, and Su (2010) solves the joint problem numerically, simulating the optimal CEO contract and under realistic assumptions on the agent’s risk aversion and actual executive contracts.

The literature on sorting has a steady history. This literature emerged separately from the adverse selection literature since models of sorting usually restrict communication or incompleteness of contracts. Lazear (1986) examined the sorting effects of fixed versus variable pay, giving predictions on which kinds of firms would offer which types of compensation contracts. Ray (2007a) explores the sorting effects of performance evaluations, finding that interim evaluations on long projects allow the firm to sort good managers from bad. While Arya and Mittendorf (2005) does allow communication between the principle and agent, the analysis examines a simplified contract (stock options) to attract managers. Finally, Lazear (2000) empirically estimates the incentive and sorting effects of performance pay, finding that the introduction of performance pay leads to a 44% increase in productivity, half of which is attributable to better incentives and half to better sorting.

The vast empirical literature of PPS of executive contracts begins with the landmark study of Jensen and Murphy (1990), that finds that CEO wealth changes $3.25 for every $1000 change in shareholder wealth. Morck, Shleifer, and Vishny (1988) join Jensen and Murphy in arguing that empirical estimates of pay performance sensitivity are in some sense “too low,” since they deviate widely from theoretical predictions. Attempts to justify these low empirical estimates on risk-aversion, such as Haubrich (1994), rely on estimating parameters of the model that are notoriously difficult to measure, such as the manager’s cost of effort parameter.3

The simple model of this paper generates a number of comparative statics that can be used to provide theoretical guidance and future empirical tests:

1. When the manager’s outside option is high, PPS increases in the variation between managers.

2. The firm will adjust salaries, but not the bonus, in response to changes in the

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3Murphy (1999) includes several stylized facts on pay performance sensitivity: PPS comes primarily through stock options and stock ownership (Hall and Liebman 1998); PPS differs across industries, and is especially low in regulating utilities and high in financial services and technology; PPS is larger in small cap industrials versus mid cap industrial companies; and PPS, on average, has increased through the 1990s. A deeper analysis of these stylized facts can test some of the comparative statics and empirical implications of this paper.
environment, such as changes in the manager’s cost of effort, the cost of employing
the manager, and the quality of match between the manager and the firm.

3. Under risk-aversion, sorting dampens PPS if the quality of the manager-firm match
is sufficiently high and production uncertainty is sufficiently low.

4. The classical negative tradeoff between risk and incentives holds if and only if the
manager’s outside option is sufficiently high.

5. As output risk increases, the firm hires better managers.

The main text of the paper provides the intuition behind each of these corollaries.
The outline of the paper is as follows: Section 2 presents the basic model of sorting
without a moral hazard problem. Section 3 combines sorting and incentives. Section 4
adds risk-aversion. Section 5 concludes.

2 The Basic Model

To fix ideas, consider the basic model with sorting, but no incentives. A firm (the
principal) employs a manager (the agent). The manager is risk neutral. The manager
has a type $\theta$, which he knows but the firm does not. The firm’s uncertainty on
$\theta$ is represented by the density $f$ with cumulative distribution function $F$, over support
$\Theta = [0, \infty)$, with mean $\mu_\theta$ and variance $\sigma_\theta^2$. The firm employs only a single manager,
and the firm’s output with a manager of type $\theta$ is

$$x = \gamma \theta + \epsilon$$

where $\epsilon$ has mean 0 and variance $\sigma^2$. The parameter $\gamma > 0$ represents the complementar-
ty between the firm and the manager. High $\gamma$ firms produce more output with high
$\theta$ managers than with low $\theta$ managers. The manager enjoys an outside option $\bar{u} > 0$,
which represents his outside opportunities. The firm bears a fixed cost $k > 0$ to employ

\[\text{Risk neutrality is a relevant benchmark that permits analysis without distortions arising from risk}
\text{aversion. Paper number 4 also operates entirely in a risk neutral framework to deliver intuition on}
\text{performance measurement and contracting.}
\]

\[\text{The manager’s outside options $\bar{u}$ are fixed and do not vary with $\theta$. This is the standard assumption}
\text{in agency models. However, this assumption is without loss, since the results here will still hold under}
\text{outside options that are increasing and linear in the manager’s type. However, to keep the analysis}
\text{simple, I maintain the standard assumption that the outside option is fixed.}
\]
the manager, which reflects training and other forms of human capital investment.

### 2.1 First Best

A social planner maximizes total surplus, which is \( TS = x - k \) for each \( \theta \). Expected surplus for each \( \theta \) is \( E[TS|\theta] = \gamma \theta - k \). Ex-post efficiency will require that total surplus is positive for each \( \theta \). This occurs when

\[
\theta > \frac{k}{\gamma} \equiv \theta_{FB}^* \tag{2}
\]

Thus, ex-post efficiency establishes a marginal type \( \theta_{FB}^* \) above which all managers of type \( \theta \) generate positive surplus. Observe that the first best cutoff \( \theta_{FB}^* \) rises in \( k \) and falls in \( \gamma \), so it is efficient for the firm to hire better managers when each manager is more expensive (to compensate for the high fixed cost of hiring), and also when the quality of the match with the firm is low (to compensate for the lower productivity from a poor match). Ex-ante surplus is

\[
E[TS] = \int_{\theta_{FB}^*}^{\infty} E[TS|\theta] f(\theta) d\theta = \int_{k/\gamma}^{\infty} (\gamma \theta - k) f(\theta) d\theta. \tag{3}
\]

The firm cannot observe the type of the manager, and therefore must induce his employment through a compensation contract. A contract is a salary \( s \geq 0 \) and a bonus \( b \geq 0 \). For tractability, I restrict attention to linear contracts of the form

\[
w = s + bx. \tag{4}
\]

This reflects the main feature of most compensation schemes that have a fixed salary and a bonus that depends on some performance measure.

Contracts are incomplete in that the firm cannot offer a menu of contracts to the manager that depends on an announcement of the manager’s type. This occurs because communication between the manager and the firm is costly, and the type \( \theta \) is sufficiently complex that it cannot be embedded within a contract. For example, \( \theta \) denotes the ability of the manager, representing a complex mix of skills and attributes, such as vision, leadership ability, efficiency of decision making, aptitude with building relations within and outside the firm, time management skills, and so on. This form of “soft” information
and “soft” skills are not contractable, yet nonetheless important for productivity.\footnote{Baker and Jorgensen (2003) impose a similar assumption on restricting communication between the firm and the agent.} And finally, observe that most CEO pay contracts do not condition on messages sent between the candidate manager and the firm. I assume that this message game does not take place, because a legal employment contract cannot condition on the soft information that characterizes the manager’s type $\theta$. This is the reason contracts are incomplete.

The timing of the game, displayed in Figure 1, runs as follows: Nature reveals $\theta$ to the manager; the firm selects a contract $(s, b)$; each manager of type $\theta$ decides whether to join the firm; Nature reveals production uncertainty $\epsilon$; and the firm pays the manager $w$ based on the realization of $x$.

\begin{figure}[h]
\centering
\begin{tabular}{lllll}
Nature & Firm proposes & Each manager & Nature resolves & Firm pays  \\
reveals $\theta$ to & contract $(s, b)$ & decides whether & $\epsilon$, and thus $x$ & manager \\
manager & & to join firm & & $w = s + b x$
\end{tabular}
\caption{Timeline of the Basic Model.}
\end{figure}

\begin{equation}
\theta^* = \frac{\bar{u} - s}{b \gamma}
\end{equation}

The marginal type $\theta^*$ depends on the contract parameters $(s, b)$ and therefore is the primary instrument through which the firm sorts managers. This sorting takes place if and only if $\theta^* > 0$, which occurs when $b > 0$ and $s \neq \bar{u}$. The sorting can either be positive or negative, in that the compensation contract can either attract all $\theta > \theta^*$ (positive sorting) or all $\theta < \theta^*$ (negative sorting). It is easy to see that if any sorting occurs at all ($\theta^* > 0$), positive sorting occurs if the manager’s expected wage
strictly increases in his type (in particular, that $s < \bar{u}$ and $b > 0$). Thus, in order to achieve positive sorting, the firm will choose a salary and bonus such that the manager’s expected wage is strictly increasing.

The marginal type $\theta^* > 0$ will still exist if the manager’s expected wage decreases in his type ($s > \bar{u}$ or $b < 0$), but the firm will never select such salaries and bonuses. The full argument rests in the proof of Proposition 1, which solves for the optimal contract, but the intuition is straightforward. The firm would never set a negative bonus because this would either attract no one (if $s < \bar{u}$) or would attract only the low types ($s \geq \bar{u}$). If the firm offers no incentive pay at all ($b = 0$), then the firm would always set $s = \bar{u}$; setting a salary below $\bar{u}$ would attract no one and setting a salary strictly above $\bar{u}$ transfers unnecessary rent to the manager. Finally, observe that for the marginal type $\theta^*$, his expected wage exactly equals his outside option ($E[w|\theta^*] = \bar{u}$), whereas every $\theta > \theta^*$ enjoys an information rent $E[w|\theta] - \bar{u} > 0$.

It may surprise you that the marginal type $\theta^*$ falls in both salary and bonus. Much of the public discourse on the sorting effects of incentive plans suggests that higher incentives attract better workers. This intuition is true, but only for very low levels of $b$. If the firm offers no incentive pay at all, then the only equilibrium is to set salary exactly equal to the manager’s outside option ($s = \bar{u}$), which attracts the entire labor market to the firm. As soon as the firm pays a positive bonus, $\theta^* > 0$ and the firm attracts higher quality candidates. In this sense, incentive pay does have a sorting effect. But as the firm continues to raise bonuses, this only transfers rents to the manager, since the bonus determines the sharing rule on the firm’s expected output. This increase in rent transfer then attracts more people to the firm, and the marginal type $\theta^*$ sinks in $b$. Indeed, because the bonus serves only a sorting effect and not an incentive effect in this benchmark model, the firm needs only to provide a nominal bonus to induce some separation, and anything beyond that nominal level will unnecessarily transfer rent to the agent. The optimal contract below further highlights this point.

2.3 Firm’s Problem

The firm earns profit from output, pays out wages, and bears a fixed cost of employing the manager, so firm profit is $\pi = x - w - k$. Thus, the ex-post expected profits for each $\theta$ is

$$E[\pi|\theta] = \gamma \theta (1 - b) - s - k$$  \hspace{1cm} (6)
Thus the firm’s profits will rise in the manager’s productivity \( \theta \) and the quality of his match with the firm \( \gamma \), but fall in the compensation parameters \( s \) and \( b \). When the firm chooses its contract \((s, b)\), this will determine the marginal type \( \theta^* \). In particular, this will induce sorting since only agents with \( \theta > \theta^* \) will choose to work at the firm. The firm therefore selects a salary and bonus to maximize its expected profits for all \( \theta > \theta^* \):

\[
\max_{s,b} \int_{\theta^*(s,b)}^{\infty} E[\pi|\theta] f(\theta) d\theta = \max_{s,b} \int_{\theta^*(s,b)}^{\infty} (\gamma \theta(1 - b) - s - k) f(\theta) d\theta. \quad (7)
\]

I write the marginal manager as \( \theta^*(s, b) \) to illustrate its dependence on the contract parameters. Even without a moral hazard problem of the manager, the compensation contract has a role to play as a sorting instrument for the firm. The contract parameters \( s \) and \( b \) will affect the firm’s payoff in two ways. First, they will determine the mix of managers attracted to the firm, and second, they will determine the firm’s expected wage payments made to every manager who then joins the firm. This dual effect of the contract (determining participation ex-ante and expected wage payments ex-post) is apparent from (7), and will be a constant theme throughout the paper, even under moral hazard and risk aversion.

Because there is no incentive problem in the basic model, the contract serves only to sort managers. Ex-post expected profit \( E[\pi|\theta] \) decreases in salary and bonus, so sorting is costly for the firm. But it is necessary, because the marginal type \( \theta^*(s, b) \) decreases in \( s \) and \( b \). Thus, as the firm raises either salary or bonus, while this decreases ex-post profits, it can conceivably raise ex-ante profits because it attracts more managers to the firm (thereby expanding the area of integration in (7)). The manager’s participation decision depends only on whether his expected profits exceed his outside options. He is effectively indifferent to receiving salary or bonus, as long as he earns more at the firm than in the outside market. The firm, however, prefers to sort using salary, rather than bonus. Intuitively, the bonus effectively establishes a sharing rule on firm output, and thus the firm prefers to make a cash transfer in salary rather than sacrifice a share of output. The firm’s problem is to attract labor in the cheapest manner possible, since the compensation contract serves only to sort managers. Because sharing output is more costly to the firm than paying cash, the firm will always substitute away from the bonus and into the salary. In fact, the firm will pay as small a bonus as possible and a salary as close to the manager’s outside option, just enough to induce him to accept the job. This leads to the first proposition (all proofs are in the appendix).
Proposition 1 *In the pure sorting model, the optimal contract is* \( s \approx \bar{u} \) *and* \( b \approx 0 \).*

In the optimal contract, the firm will set a salary arbitrarily close to \( \bar{u} \) and a bonus arbitrarily close to 0. The firm does not set \( s = \bar{u} \) and \( b = 0 \) exactly, because then *every* manager \( \theta \) would weakly prefer to work at the firm. But the firm could do strictly better by reducing the salary by an arbitrarily small amount and raising the bonus by an arbitrarily small amount. That will provide just enough steepness to the wage schedule, such that only \( \theta > \theta^{FB} \) prefer to work at the firm. The proof of Proposition 1 shows that such a contract dominates \( s = \bar{u} \) and \( b = 0 \) and achieves efficient sorting.

\[\text{Figure 2: Equilibrium in the Benchmark Model}\]

\[\text{At a technical level, } \theta^* \text{ is undefined at the point } s = \bar{u} \text{ and } b = 0. \text{ The firm’s maximization problem in (1) therefore has a discontinuity at the point } (s, b) = (\bar{u}, 0). \text{ The firm’s expected payout at this discontinuity is strictly less than the payoff from the contract } s = \bar{u} - k\eta \text{ and } b = \eta \text{ for sufficiently small } \eta > 0. \text{ Thus the optimal contract is arbitrarily close, but not exactly equal to } (s, b) = (\bar{u}, 0). \text{ In practice, take } \eta \text{ to be the smallest possible value in the available currency, such as one cent.}\]
To see this visually refer to Figure 2. The firm has two contract parameters, salary and bonus, to induce the sole decision of the agent (participation). Therefore there are many, in fact a continuum, of contracts that can induce efficient participation. The heavy line in Figure 4 represents the efficient set, namely, contracts \((s, b)\) that induce efficient participation: \(\{(s, b) : \theta^*(s, b) = \theta^F B\}\). Observe that the efficient set slopes downward, so salary and bonus are substitutes rather than complements in inducing participation. This shows that participation is truly a bivariate rather than univariate problem. The contract is a pair, and the firm selects each parameter jointly rather than individually. Raising the bonus alone will attract more (and worse) managers, since large incentive payments increase everyone’s wages. The firm can compensate for this by lowering salary, bringing participation back to its efficient level. In this sense, the mix of compensation is as relevant, if not more relevant, than simply the level of compensation.

Figure 4 also graphs the isoprofit lines of the firm. These are the isoquants of the firm’s profit function given in (7). Simple algebra shows that these isoprofit lines must be steeper than the slope of the efficient set. And as the fixed level of profit rises, the isoprofit lines shift downward. Thus, the firm will choose the contract with an isoprofit line that gives maximum profit and still achieves efficiency. Because of the differential slope of the two linear lines (the isoprofit line and the efficient set), this happens exactly at the corner solution where the two lines intersect.

\[
E[\pi | \theta] = \gamma(1 - b)E[\theta | \theta > \theta^*] - s - k)(1 - F(\theta^*)) \tag{8}
\]

Proposition 1 shows that performance pay does indeed have a sorting effect, but that this effect operates at very low pay-performance sensitivities. All that is needed to induce efficient sorting is a non-zero slope of the wage profile. More than this is unnecessary because it transfers unnecessary rent to the agent and erodes the firm’s share of output. Moreover, the sorting effect exerts downward pressure on performance pay. Performance pay attracts better workers, but excessive performance pay actually attracts worse workers and is costly for the firm. This benchmark model of pure sorting establishes this downward pressure on PPS from the sorting effect. Next I embellish the model to include incentives to explore how this sorting effect interacts with the incentive effect of the compensation contract.

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8Fixing ex-post profits at a given level and rewriting that equation as salary as a function of the bonus delivers the dashed isoprofit lines in Figure 2.
3 Combining Sorting and Incentives

Now suppose the manager exerts costly and unobservable effort at the firm. This induces a moral hazard problem on the part of the manager, since the firm cannot observe effort perfectly, but must induce it through its compensation contract. At the same time, this same compensation contract is used to attract managers to the firm. Thus, contracts will now serve the dual purpose of attracting workers and providing incentives. Output is now given by

\[ x = \gamma \theta e + \epsilon, \]  

(9)

where \( \epsilon \) still has mean 0 and variance \( \sigma^2 \). The manager exerts effort \( e \) at a quadratic cost of effort at \( C(e) = 0.5ce^2 \) with \( c > 0 \), and he continues to enjoy an outside option \( \bar{u} \). Observe that the manager’s type \( \theta \) and effort choice \( e \) are complements, so more able managers have higher marginal productivities of labor and are therefore more productive to the firm. In fact, there are two levels of complementarity: between the ability \( \theta \) and effort \( e \), as well as between ability and the quality of the match between the firm and worker \( \gamma \). In practice, \( \gamma \) is general to the firm and all the workers within the firm, whereas \( \theta \) and \( e \) are specific to each individual manager. Of course, \( e \) is a choice variable, \( \theta \) is a random variable, and \( \gamma \) is an exogenous parameter.

The firm pays the manager \( w = s + bx \). As before, the contract is linear and the firm cannot condition the contract on the manager’s type. Observe that the expected output from the firm for each manager of type \( \theta \) is \( E[x|\theta] = \gamma \theta e \), so more effort from the manager produces more revenue for the firm. For each \( \theta \), the average wage is \( E[w|\theta] = s + b\gamma \theta e \). Assume the manager is risk-neutral.\(^9\) Figure 3 illustrates the timeline of the game. Nature reveals \( \theta \) to the manager (but not the firm); the firm proposes a contract \((s, b)\); each manager of type \( \theta \) decides whether to join the firm; managers who join exert effort \( e \), while managers who do not join collect their outside option \( \bar{u} \); Nature resolves production uncertainty \( \epsilon \), and this determines output \( x \); the firm pays the manager wages \( w = s + bx \) based on output \( x \).

\(^9\)The next section considers a risk averse manager.
<table>
<thead>
<tr>
<th>Nature reveals θ to manager</th>
<th>Firm proposes contract (s, b)</th>
<th>Each manager decides whether to join firm</th>
<th>Managers who join exert effort e, and thus x</th>
<th>Nature resolves manager</th>
<th>Firm pays manager</th>
</tr>
</thead>
</table>

Figure 3: Timeline of the Game with Effort.

### 3.1 First Best

The manager has two decisions: whether to join the firm and how hard to work. As such, the first best benchmark will also have two components, an efficient effort level and an efficient cutoff for participation. Observe that the expected total surplus for each θ is

\[
E[TS|θ] = γθe - C(e) - k.
\]

Total surplus now not only includes expected output and the fixed cost of hiring a manager, but also the manager’s cost of effort. The first best effort level that maximizes the expression above is \(e_{FB} = γθ/c\). Observe that the first best effort level rises in both \(θ\) as well as in \(γ\). It is efficient for more productive managers to exert more effort, where productivity is measured in either a manager-specific sense (θ) or a firm-specific sense (γ). Naturally, it is efficient for managers with high cost of effort to exert less effort, since this high cost reduces the manager’s utility and therefore total surplus as well.

Evaluated at \(e_{FB}\),

\[
E[TS|θ] = \frac{(γθ)^2}{2c} - k.
\]

Expected total surplus rises in both \(γ\) and \(θ\), and falls in the cost of effort parameter \(c\) and the fixed cost of hiring \(k\). This is positive if and only if

\[
θ > \frac{\sqrt{2ck}}{γ} ≡ θ_{FB}.
\]

As before, \(θ_{FB}\) denotes an efficient cutoff, namely the minimal managerial type such that it is efficient for the firm to employ any manager with \(θ > θ_{FB}\). The cutoff \(θ_{FB}\) rises in the cost of the manager \(k\) and falls in the measure of complementarity \(γ\). But observe that \(θ_{FB}\) also rises in the manager’s cost of effort parameter \(c\). Thus, as technological
or market factors lower the cost of supplying effort of all managers, it is efficient for the firm to hire more managers, and have each of them work more.

### 3.2 Manager’s Problem

Each manager of type $\theta$ maximizes his expected wage less his cost of effort:

$$ \max_e E[w|\theta] - C(e) $$

The first order condition yields the manager’s incentive constraint (IC): $\hat{e} = b\gamma \theta / c$. Higher bonuses now have a clear incentive effect of inducing more effort. In addition, effort rises in both $\gamma$ and $\theta$, so more able managers work more, as do managers who are a better fit with the firm. This is exactly the sense in which there is complementarity in production: both $\gamma$ and $\theta$ are complements with respect to effort.

The participation decision now involves the manager’s effort choice, which she makes conditional on facing a contract $(s, b)$. A manager of type $\theta$ will join the firm if in equilibrium, the manager earns more inside the firm than outside the firm, which occurs if $E[\hat{w}|\theta] - C(\hat{e}) \geq \bar{u}$. The manager’s incentive constraint gives equilibrium effort $\hat{e} = b\gamma \theta / c$, so his equilibrium cost of effort is $C(\hat{e}) = (b\gamma \theta)^2 / 2c$, and his equilibrium wage is $E[\hat{w}|\theta] = s + b\gamma \theta \hat{e} = s + (b\gamma \theta)^2 / c$. Thus, there exists a marginal manager $\theta^*$ that satisfies $E[\hat{w}|\theta^*] - C(\hat{e}) = \bar{u}$, or

$$ \theta^* = \sqrt{\frac{2c(\bar{u} - s)}{b\gamma}}. $$

As before, the marginal manager $\theta^*$ falls in both $s$ and $b$, reinforcing the intuition that higher wage payments attract more managers to the firm. And just as with $\theta^{FB}$, $\theta^*$ rises in $c$. The firm will achieve positive sorting ($\theta > \theta^*$) if the expected wage profile is strictly increasing in $\theta$. This holds if $s < \bar{u}$ and $b > 0$, just as before. By similar arguments, the firm will never set $s > \bar{u}$ or $b < 0$.

### 3.3 Impediments to First Best

Can the firm implement first best? Presumably, this may be possible because there is no difference in risk preferences between the firm and the agent. The canonical agency model of a risk neutral principal and agent obtains first best. However, sorting complicates the analysis.
Expected profits of the firm are \( E[\pi|\theta] = E[x|\theta] - E[w|\theta] - k. \) Plugging in (IC),

\[
E[\pi|\theta] = (\gamma \theta)^2 b(1 - b)/c - s - k. \tag{15}
\]

As before, the firm’s profits rise in the complementarity parameters \( \gamma \) and \( \theta \) and fall in the cost of effort parameter \( c \) and the fixed cost of hiring \( k \). However, the effect of the bonus on profits is more subtle even though paying out salary clearly drains profits. In particular, a small bonus will raise expected profits, but as the bonus becomes very large, it will eventually lower the firm’s profits. This occurs because of the dual sorting and incentive effects.

Comparing the efficient effort \( e^{FB} \) and the efficient cutoff \( \theta^{FB} \) to the manager’s equilibrium effort \( \hat{e} \) and type \( \theta^* \) reveals that the firm could implement first best by setting \( b = 1 \) and \( s = \bar{u} - k \). Intuitively, the firm grants maximum incentives to the agent in order to induce him to exert the efficient effort level, and will set a salary at the manager’s outside option less his cost of employment, thereby inducing efficient participation. Because the firm has two instruments (salary and bonus) to solve two problems (effort and participation), it seems plausible that the firm could implement first best. However, observe that at the proposed contract \( (s, b) = (\bar{u} - k, 1) \), the expected profit of the firm is \( E[\pi|\theta] = -s - k = -\bar{u} < 0 \). Thus, to implement first best, the firm goes bankrupt. Even though the firm has two separate instruments to solve the effort and participation decisions, this is not sufficient to induce efficiency.

To understand this, recall the canonical agency problem of a principal contracting with a risk-neutral agent.\(^\text{10}\) In that setup, the principal can induce first best effort by a “sell-the-firm” contract, in which he makes the agent the full residual claimant on output, and induces participation by setting a salary sufficiently low. In equilibrium, the agent earns exactly his outside option, and the principal earns profit from the trade by extracting the rent through the low salary (in the extreme case of no outside option, the salary is negative). In the canonical model, the principal solves the incentive problem with the bonus coefficient and induces participation with the salary. Because of the simple nature of the participation decision in the canonical model, the salary both equilibrates the agent’s payoff to his outside option, and transfers surplus back to the principal.

In the richer sorting model here, the salary cannot simultaneously solve the partici-

\(^\text{10}\)For completeness, I provide the canonical model in Appendix B.
pation problem and transfer rent to the firm. To see this, observe that if the firm seeks to induce sufficient effort, then it must set $b = 1$, and therefore its expected profit is $E[\pi|\theta] = -s - k$, which is positive if and only if $s < -k$. But in order to achieve efficient participation ($\theta^* = \theta^{FB}$), it must be the case that $s = \bar{u} - k > -k$. Therefore, the firm must choose between achieving efficient participation and staying solvent. It cannot do both. The richer structure of the participation problem in this paper, relative to the canonical model, constrains the ability of the salary to guarantee efficiency. In effect, the participation problem interferes with the firm’s solvency constraint, while this is not true in the canonical model.

### 3.4 The Firm’s Problem: Second Best Solution

Now that it’s clear the firm cannot achieve first best, it will select a salary and bonus to maximize its expected profits for every manager who joins the firm. Thus, the firm will select $s$ and $b$ to maximize

$$\Pi(s, b) = \int_{\theta^*(s,b)}^{\infty} E[\pi|\theta] f(\theta) d\theta. \quad (16)$$

As before, $\theta^*(s, b)$ makes prominent the dependence of the marginal manager on the parameters of the compensation contract that the firm sets. Because of the moral hazard problem, the compensation contract plays a dual role of both sorting managers through $\theta^*$, as well as providing incentives to managers through the expected wage $E[w|\theta]$. Solving this program explicitly for $s$ and $b$ is difficult because of the interaction between the sorting and incentive effects. In particular, $s$ and $b$ will both affect the manager’s incentives to work, conditional on participating, as well as determine whether he chooses to participate at all. Thus, the participation decision $\theta^*(s, b)$ is now endogenous. Nonetheless, the next proposition, proved in the appendix, presents the implicit solution that still has enough structure to provide insight into the dual sorting and incentive effects.

**Proposition 2** A firm contracting with a risk neutral agent will select an optimal contract $(s, b)$ that satisfies

$$s = \left[ \theta^*(1 - b) - \frac{1 - F(\theta^*)}{f(\theta^*)} b \right] \frac{b\theta^*c^2 - k}{c} \quad (17)$$

16
\[ b = \left( 2 - \frac{\theta^*^2}{E[\theta^2|\theta > \theta^*]} \right)^{-1}. \]  

(18)

The sorting effect is immediately apparent in the optimal contract, since the contract now depends on the distribution of manager types. Recall that in the canonical agency problem of a principal contracting with a risk-neutral agent, the principal will set a salary equal to the agent’s outside option and make the agent the full residual claimant on the firm’s output \( (b^* = 1) \); this is the “sell-the-firm” contract, in which the agent keeps the entire share of firm output, and the principal takes back those rents in expectation via the (possibly negative) salary. The canonical model has empirical difficulties both because negative salaries are uncommon, and because empirical estimates of pay-performance sensitivities are much lower than \( b = 1 \).

Adding sorting to the canonical model however, moves the equilibrium away from the “sell-the-firm” contract. For all \( \theta > \theta^* \), observe \( E[\theta^2|\theta > \theta^*] > \theta^*^2 \), and therefore, \( b < 1 \). Thus, the presence of sorting once again dampens the optimal bonus. This confirms the intuition from Proposition 1 that while a positive bonus is necessary to attract higher quality managers to the firm, too much weight on this bonus is wasteful because it transfers excessive rent to the agent. Here, the firm must provide incentives to the manager in order to induce him to work, which puts upward pressure on the bonus. The firm must also attract quality managers and yet this transfers rent to the managers and places downward pressure on the bonuses. The optimal bonus will trade off these twin effects, namely the downward pressure from sorting and the upward pressure from incentives.

The key term that governs the behavior of the optimal bonus is \( \theta^*^2 / E[\theta^2|\theta > \theta^*] \). Call this the marginal-inframarginal ratio (MIR). Namely, the ratio between the marginal type \( \theta^*^2 \) and the inframarginal type \( E[\theta^2|\theta > \theta^*] \). The marginal manager \( \theta^* \) is indifferent between accepting and rejecting the contract, while the inframarginal types are all those managers who have ability greater than \( \theta^* \) and choose to accept the contract. This ratio effectively measures the need for sorting. To see this, observe that when the distribution \( f \) places a large mass in the tails of the distribution, then \( E[\theta^2|\theta > \theta^*] > \theta^*^2 \), and therefore the MIR is small. This means that the expectation of the inframarginal types far exceeds the marginal type. The firm needs only a small bonus in order to attract this large pool of managers. In contrast, when the distribution of \( f \) is concentrated such that there is little probability mass beyond \( \theta^* \), \( E[\theta^2|\theta > \theta^*] = \theta^*^2 \) and the MIR is large, close
to 1 (because \(E[\theta^2|\theta > \theta^*] > \theta^*^2\)). The firm will choose a larger bonus in order to induce this small pool of managers to the firm. Because the MIR fundamentally depends on the distribution of types, this provides intuition on how the optimal bonus also depends on the distribution of types. Paying a share of output through a bonus is expensive for the firm, but the firm will do so if it needs to induce managers to work. This occurs precisely when the pool of potential managers is small, and therefore the MIR is high.

How exactly will the firm trade off the optimal choice of salary and bonus? In the canonical model of risk-neutral agent without sorting, there is a clean separation between salary and bonus; the bonus provides incentives, while the salary guarantees participation. Here, though the salary does not play a role in incentives, bonuses do affect sorting, since the marginal manager \(\theta^*\) falls in \(b\). Just like salaries, higher bonuses will attract more managers to the firm. The proof of Proposition 2 details the first order condition for the firm’s optimization with respect to the optimal salary and bonus, which leads to the equilibrium condition

\[
\frac{\int_{\theta^*}^{\infty} \partial E[\pi|\theta]/\partial s \ dF}{\int_{\theta^*}^{\infty} \partial E[\pi|\theta]/\partial b \ dF} = \frac{\partial \theta^*/\partial s}{\partial \theta^*/\partial b}.
\] (19)

This equilibrium condition is the marginal rate of substitution between salary and bonus. Recall from Section 2 that sorting gives contracts a dual effect, namely determining participation ex-ante and the expected wage payments ex-post. The same is true here, after including a moral hazard problem. Each piece of the compensation contract will have an ex-post effect (left-hand side of (19)) and an ex-ante effect (right-hand side of (19)). The ex-ante effect is on \(\theta^*\), and specifies how the contract parameter changes the marginal manager (\(\partial \theta^*/\partial s\) and \(\partial \theta^*/\partial b\)), thereby determining the participation decision. The ex-post effect is on ex-post profits \(E[\pi|\theta]\) for each \(\theta > \theta^*\), and determines the cost of the contracts to the firm for every manager that the firm hires. This effect is \(\frac{\partial E[\pi|\theta]}{\partial s}\) for each \(\theta > \theta^*\), and similarly \(\frac{\partial E[\pi|\theta]}{\partial b}\) for each \(\theta > \theta^*\). Each contract parameter exerts these twin effects, the ex-ante effect on participation and the ex-post effect on wage costs. The equilibrium condition above states that the firm will optimally equalize the ratio of ex-ante and ex-post effects of the salary and bonus. This is equivalent to equalizing the marginal rate of substitution between salary and bonus. The firm chooses its salary and bonus such that the tradeoff between the costs of wages against the benefits of participation is equal across both contract parameters. At the equilibrium, the firm is indifferent between using salary and bonus because their marginal rates of substitution
are equal.

To see this visually, refer to Figure 4. First, observe that the existence of both a sorting and incentive problem implies a multidimensional measure of efficiency: there is now an efficient participation level \( \theta^{FB} \) as well as an efficient effort level \( e^{FB} \). Because the firm has two contract parameters (salary and bonus) to solve the participation and effort problems, the efficient set now collapses to a single point, rather than a continuum as in the benchmark model. Figure 4 plots the efficient contract \((s, b) = (\bar{u} - k, 1)\). The firm’s expected profit function is

\[
\Pi(s, b) = (1 - F(\theta^*)) \left( \frac{\gamma^2}{c} b(1 - b) E[\theta^2 | \theta > \theta^*] - s - k \right) \quad (20)
\]

Observe that this is the expected profit as a function of the contract parameters \((s, b)\). Rewriting this expression as the salary as a function of the bonus (for a fixed level of expected profits) gives the isoprofit curve pictured as the upside down U-shaped hyperbole in Figure 4. This contour line gives the set of salary and bonus pairs that guarantee a fixed level of profit. As that fixed level increases, the intercept for this hyperbole shifts down and the curve moves downward. This is exactly why the firm cannot implement efficiency, since the isoquant that contains the efficient contract would require an upward shift of the isoquant, possible only with a negative profit level. Thus, the efficient contract sits above and outside of the isoprofit line of the profit maximizing contract.

The other curve in Figure 4 is the isoquant for the marginal manager \( \theta^* \) given by (12). The isoquant graphs the contract parameters that induce a fixed level of participation, and the downward sloping shape of this isoquant confirms that salary and bonus are substitutes. They are different but equivalent instruments to inducing participation, as an increase in the bonus will require a decrease in the salary in order to keep participation fixed. This reinforces the point that participation is truly a bivariate and not a univariate phenomenon, and that the firm must set the choice of salary and bonus jointly rather than individually to understand their effect on participation. If the firm wants to induce a higher level of participation \( \theta^* \), then the isoquant in Figure 4 will tilt leftward.\(^11\)

The isoprofit and isoparticipation lines represent the ex-post and ex-ante effects of the contract, respectfully. A given contract determines not only who participates ex-

\(^11\)In particular, the y-intercept of the isoquant remains fixed at \( \bar{u} \) while the x-intercept shrinks, shifting leftward.
ante, but also how hard the manager works ex-post. The slope of each isoquant is the marginal rate of substitution between salary and bonus. The equilibrium condition shows that this occurs precisely when the marginal rates of substitution between salary and bonus are equivalent for both the ex-ante and ex-post effects. This occurs precisely when the two isoquants have identical slope. Because the isoparticipation line is downward sloping, this will necessarily occur on the downward sloping portion of the isoprofit line.

\[ \theta^* = \sqrt{2c(\bar{u} - s)/b\gamma} \]

\[ \Pi = \int_{\theta^*}^{\infty} E[\pi|\theta]dF \]

Figure 4: Equilibrium in the Combined Incentives and Sorting Model

Recall that in the benchmark model, the firm always prefers to substitute salary for bonuses to induce sorting. This is precisely because there is no incentive problem in the benchmark model. Without moral hazard issues, the firm need not induce effort, and therefore, paying bonuses simply drains expected profit. Thus, the equilibrium condition above collapses to a corner solution in which the firm pays as small a bonus as possible to guarantee participation and positive sorting. Adding an effort problem to the manager’s decision gives a positive role for the bonus. The firm’s expected profit
function is concave in the bonus, and this concavity gives an interior solution to the equilibrium condition above. Now, the firm’s choice of bonus not only draws managers to the firm, but also provides incentives to the managers once they accept the job. This extra incentive effect of the bonus is what gives a non-trivial solution for the optimal contract, as detailed in Proposition 2.

3.5 Comparative Statics and Empirical Implications

Because the optimal contract serves a sorting role in addition to incentives, the optimal salary and bonus both depend on the distribution of managerial talent \( f \). Despite the lack of an explicit closed-form solution, the optimal contract for Proposition 2 easily delivers the following corollary.

**Corollary 1** When cost of effort \((c)\) or the outside option \((\bar{u})\) are sufficiently large, the PPS increases in the variation between managers.

The proof of this corollary assumes for tractability that the distribution of managers is uniform, so \( f(\theta) = 1/2a \) for \( a > 0 \). Here, the parameter \( a \) tracks the support of the uniform distribution, as well as the variance of \( \theta \). The intuition for the proposition stems from the marginal-inframarginal rate (MIR), \( \theta^*/E[\theta^2|\theta > \theta^*] \). Under a uniform density, the proof of Corollary 1 shows that this is

\[
MIR = \frac{3\mu\theta^* + 3a\theta^* - \theta^*}{(\mu + a)^3 - \theta^*^3}
\]  

(21)

When either the cost of effort \((c)\) or the managers outside options \((\bar{u})\) are sufficiently large, the marginal manager \( \theta^* \) is large, and MIR then increases in \( a \), thereby causing the optimal bonus \( b \) to also increase in \( a \). Intuitively, when either the workers’ cost of effort or outside options are high, this inflates the marginal manager \( \theta^* \), which also inflates MIR. As argued earlier, when MIR is high the distribution places relatively low mass on managers \( \theta > \theta^* \), and so the firm needs to offer a high bonus in order to induce this small pool of managers to accept the job. Conversely, a small cost of effort or outside option leads to a low \( \theta^* \) which sinks MIR, thereby revealing a large pool of workers with \( \theta > \theta^* \). This allows the firm to offer a small bonus to induce this large pool of managers to accept the job.

This corollary confirms the results of Dutta (2008) and Baker-Jorgenson (2003), but for different reasons. Both papers find in their models that their optimal bonus can
rise in what Dutta (2008) calls “information risk,” namely, the variance on \( \theta \). For Baker-Jorgenson, the firm wants the manager to make use of his private information and increasing the bonus induces the manager to make use of this information when the value of that information rises when the variance of \( \theta \) rises. Dutta (2008) arrives at the same result but does so because the manager’s outside option increases in his ability \( \theta \), thereby inducing the firm to choose a higher bonus when the variance of \( \theta \) is high. I advance an alternative explanation here based on sorting and the tradeoff with incentives. Incentive provision is costly for the firm, and so the firm prefers to minimize bonus payments. But the firm will raise the bonus when necessary when there is a strong need to induce participation. This occurs exactly when MIR is large, which itself occurs when \( a \) is large under a high \( \theta^* \).

**Corollary 2** *As the manager’s cost of effort (c) rises, cost of employment (k) rises, and complementarity with the firm (\( \gamma \)) falls, the firm will lower salaries but leave the bonus unchanged.*

This corollary follows directly from inspection of the optimal contract in Proposition 2. To gain insight on the underlying economic forces, observe that the first order condition from the firm’s choice of salary is

\[
E[\pi|\theta^*]f(\theta^*) \frac{\partial \theta^*}{\partial s} = \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial s} f(\theta) d\theta
\]  

(22)

This first order condition equates the marginal cost and marginal benefit of adjusting the salary. The left-hand side is the marginal benefit of a salary change. The firm earns an expected profit of \( E[\pi|\theta^*] \) for the marginal manager \( \theta^* \), multiplied by the point probability \( f(\theta^*) \). The firm earns this profit on the marginal manager, and \( \frac{\partial \theta^*}{\partial s} \) captures the change in the marginal manager from the incremental change in salary. Thus, a small change in salary will change the marginal manager, and this will yield a return to the firm measured by the expected profit of that marginal manager. The right-hand side of (22) captures the marginal cost of an incremental salary change. The derivative inside the integral is the incremental impact on firm profits for an incremental change in salary. Because the firm pays the same salary to all managers it hires, the marginal cost is the sum (integral) of this incremental effect on profit for every \( \theta > \theta^* \). In equilibrium,
the marginal cost and marginal benefit of this incremental change in salary are equal.\footnote{Because the salary has no incentive effects, the change in profit from a change in salary is simply $-1$, so \( \frac{\partial E[\pi|\theta^*]}{\partial s} = 1 \). Thus, a $1 increase in salary drains firm profit by exactly $1. Thus, (22) simplifies to
\[
E[\pi|\theta^*]H(\theta^*) \frac{\partial \theta^*}{\partial s} = 1 \tag{23}
\]
where \( H(\theta^*) = f(\theta^*)/(1 - F(\theta^*)) \) is the hazard rate, familiar from the literature on adverse selection models.}

The firm selects the salary such that (22) holds in equilibrium. Consider the effect of increasing the cost of employing and training the manager \( k \). The right-hand side of (22) is unchanged, but increasing \( k \) lowers expected profits. Because the right-hand side of (22) is independent of \( k \), and because the equality must hold in equilibrium, the firm will compensate for this reduction in expected profit by lowering the salary, which effectively lifts expected profits. Thus, the firm exerts a countervailing effect on expected profit to respond to an exogenous rise in the cost of employment. A similar effect holds with an increase in the manager’s cost of effort \( c \). The effect is slightly more subtle because the cost of effort parameter \( c \) affects the participation decision through \( \frac{\partial \theta^*}{\partial s} \), but the logic is identical. Finally, the same effect operates for a change in \( \gamma \), though the comparative static moves in the opposite direction. In sum, managers who are expensive to train, have a high cost of effort, and are a poor match with the firm, will all receive lower salaries.

It is noteworthy that the salary will absorb all of the variation in the exogenous parameters \( c, k, \) and \( \gamma \), while the bonus absorbs none of it. This predicts empirically that one would expect higher variation in salaries than in bonus contracts. Even though the salary and bonus simultaneously determine participation (\( \theta^* \) is a function of both \( s \) and \( b \)), the manager’s incentive constraint shows that only the bonus affects the effort problem. The bonus is thus doing “double duty” by determining both participation and effort, and therefore the firm uses the salary to handle variations in the exogenous parameters.

### 4 Risk Aversion

Now suppose the manager is risk-averse, and dislikes volatility in his income. Without sorting, the canonical agency model with a risk averse agent delivers the standard risk-incentives trade-off where the principal seeks to provide incentives for the agent to work,
but such incentives load risk onto the agent which he dislikes. How does sorting affect this analysis?

To fix ideas, suppose production uncertainty $\epsilon$ is normally distributed, and as before, has a mean of 0 and a variance of $\sigma^2$. Assume the agent has CARA preferences with coefficient of absolute risk aversion $r > 0$. The timing of the game is the same as before, as are the first best effort level $e^{FB}$ and first best participation cutoff $\theta^{FB}$.

### 4.1 Manager’s Problem

Given output $x = \gamma\theta e + \epsilon$ and a linear compensation contract $w = s + bx$, the certainty equivalent of the $\theta$-manager’s payoff is now

$$CE(\theta) = s + b\gamma\theta e - \frac{r^2}{2}b^2\sigma^2 - C(e).$$

(24)

Observe that uncertainty in production $\sigma^2$ affects the wage variance $V[w|\theta] = b^2\sigma^2$, but does not affect the mean $E[w|\theta] = s + b\gamma\theta e$. Therefore, the manager’s effort choice optimizes his expected wage less his cost of effort. Given a quadratic cost of effort $C(e) = 0.5ce^2$, the incentive constraint (IC) for the manager of type $\theta$ is the same as before: $e = b\gamma\theta/c$. Given a contract $(s, b)$, a manager of type $\theta$ will participate if the certainty equivalent of his contract exceeds his outside option $\bar{u}$, or $CE(\theta) \geq \bar{u}$. As before, there exists a marginal manager $\theta^*$ who is indifferent between accepting and rejecting the compensation contract. All managers of higher type $\theta > \theta^*$ will accept. This $\theta^*$ is

$$\theta^* = \frac{\sqrt{2c(\bar{u} - s) + crb^2\sigma^2}}{b\gamma}.$$

(25)

Observe that a risk-averse manager now bears a disutility from uncertainty captured by the risk premium term $\frac{r^2}{2}b^2\sigma^2$. This risk premium is the additional compensation necessary to induce a risk-averse manager to accept risk. Indeed, only a risk-averse manager of higher $\theta$ will be indifferent between accepting and rejecting the contract. Thus, the marginal type $\theta^*$ rises in the measure of the agent’s risk aversion. A risk-averse manager requires an additional risk premium to equalize his expected utility with that of a risk-neutral manager. This is true in general for each $\theta$, and in particular is true for $\theta^*$.

The firm can achieve positive sorting if $\theta^* > 0$. This occurs if $b > 0$ and $s < \bar{u} + 0.5rb^2\sigma^2$. Namely, the bonus must be positive and the salary must not exceed the
outside option plus the risk premium. Observe that this is a looser condition on the salary than before, since without risk aversion, \( s < \bar{u} \) was the sufficient condition to guarantee positive sorting. The risk premium thus grants the firm slightly more leeway in setting its salary, since the firm must embed within the salary the manager’s risk premium. Consistent with earlier results, \( \theta^* \) falls in \( s \) and \( b \), and rises in the outside option \( \bar{u} \). And if \( s < \bar{u} \), \( \theta^* \) rises in the cost of effort parameter \( c \), reinforcing the natural intuition that as effort becomes more costly, fewer managers choose to work at the firm.

The other new comparative static in this model of risk aversion is how participation changes with output volatility. As \( \sigma^2 \) rises, this loads volatility onto the manager’s compensation, which he dislikes because of his risk aversion. For a given \( \theta \), this lowers the manager’s certainty equivalent of his payoff, and therefore makes him less likely to work at the firm. Said differently, only a manager with a high \( \theta \) will generate enough revenue for the firm, and therefore also enough incentive compensation for the additional decrease in utility from the rise in volatility. Thus, high productivity managers will select the firm, whereas low productivity managers will select their respective outside options. The marginal manager \( \theta^* \), who is indifferent between his inside and outside options, will therefore increase. Raising output volatility will not only cause fewer managers to participate, but better managers to participate. Insofar as it is possible to measure production uncertainty \( \sigma^2 \) and managerial type \( \theta \), this predicts that better managers work in more uncertain environments.

The presence of the risk premium in the formula for the marginal manager \( \theta^* \) complicates the relationship between \( \theta^* \) and \( b \). In the benchmark model, \( \theta^* \) uniformly decreased in \( b \), but now the bonus appears in both the numerator and denominator in the expression for \( \theta^* \). Simple algebra shows \( \theta^* \) increases in \( b \) if and only if the manager’s salary exceeds her outside option \( (s > \bar{u}) \). Thus, when the firm sets a salary below the manager’s outside option, increasing the manager’s bonus transfers rents to the manager, thereby attracting more managers to the firm. But if the firm pays the manager an efficiency wage (where the salary exceeds the outside option), increasing the bonus lifts the risk premium so much that the rise in \( \theta^* \) due to the risk premium outweighs the fall in \( \theta^* \) due to the rent transfer (the numerator effect exceeds the denominator effect in the formula for \( \theta^* \)). Increases in \( \theta^* \) correspond to higher standards, and thus better managers come to the firm; likewise, decreases in \( \theta^* \) tend to attract worse managers attracted to the firm. Thus, the presence of risk aversion illustrates that the sorting effects of compensation are subtle. How the firm sets salary will affect the sign of \( \partial \theta^*/\partial b \), i.e.
whether sorting has a positive or negative effect on the quality of managers attracted to the firm.

To understand how the firm will optimally select its salary and bonus, now turn to the firm’s problem.

### 4.2 Firm’s Problem

The firm maximizes expected profit $E[\pi|\theta] = E[x|\theta] - E[w|\theta] - k$ for each $\theta > \theta^*$ that the firm hires. Using (IC), the firm’s expected profit, conditional on $\theta$, is

$$E[\pi|\theta] = (\gamma \theta)^2 b (1 - b)/c - s - k. \quad (26)$$

Observe that the form for expected profit is the same as before, namely, in the prior section with a risk-neutral agent. This occurs because risk aversion does not alter the agent’s incentive constraint, but does affect his participation decision. Because the incentive constraint is the same, conditional on hiring a manager $\theta$, the firm’s ex-post profits from that manager are unchanged in the presence of risk aversion. For sure, risk aversion will affect the optimal bonus, which in turn will drive both incentives and participation. Taking this bonus as exogenous, as the firm does prior to its optimization, risk aversion does not change the expected profit function.

The problems with implementing first best are similar in nature, and even more severe than in the prior section with a risk-neutral manager. In order to achieve efficient effort, the firm must make the manager the full residual claimant on firm output. With $b = 1$, the firm can induce efficient participation by setting $a = \bar{u} + \frac{c}{2} \sigma^2 - k$. To guarantee participation, the firm must compensate the manager with a base wage of at least his outside option, less the cost of his employment, plus his risk premium, to induce the risk-averse manager to accept the risky compensation scheme. However, the firm’s expected profit at this contract is $E[\pi|\theta] = -\bar{u} - \frac{c}{2} \sigma^2 < 0$. Firm profit is even lower than before because of this additional risk premium, which must be a component of the salary. As before, the firm cannot simultaneously achieve efficient participation and avoid bankruptcy.

Thus, the firm will select $s$ and $b$ to maximize profits, which are $E[\pi|\theta]$ for each $\theta > \theta^*$:

$$\Pi(s, b) = \int_{s, b}^{\infty} E[\pi|\theta] f(\theta) d\theta. \quad (27)$$
The firm will select its salary and bonus to maximize this expression. Just as in the prior model with the risk-neutral agent, optimizing jointly over \( s \) and \( b \) will lead to an equilibrium condition that equalizes the marginal rates of substitution between salary and bonus. This substitution trades off the ex-ante effect on participation against the ex-post effect on expected profits. The presence of sorting guarantees that this equilibrium condition is non-trivial. The salary and bonus will each receive non-zero weight at the optimum, as each instrument has different effects on both participation and on ex-post wages and profits. The equilibrium condition is

\[
\int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial s} \frac{dF}{\partial \theta^* / \partial s} = \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial b} \frac{dF}{\partial \theta^* / \partial b}.
\]

While the equilibrium condition takes the same form as before, it does have a few key differences. The main similarity is in the form of the expected profit function, which as argued earlier, is identical to the expression under a risk-neutral agent. Therefore, the numerator of the equilibrium condition is unchanged. What has changed is the denominator. In particular, the only term that is changed is the expression \( \frac{\partial \theta^*}{\partial b} \), which now is

\[
\frac{\partial \theta^*}{\partial b} = \frac{rc\sigma^2}{2b\theta^*} - \frac{\theta^*}{b} = \frac{rc\sigma^2}{\gamma^2b\theta^*} - \frac{\theta^*}{b}.
\]

This expression makes the effect of risk aversion clear. Under risk neutrality, \( r = 0 \) and this derivative becomes unambiguously negative, just like \( \frac{\partial \theta^*}{\partial s} \). Increasing salary and bonus raises the compensation of the agent, making the contract more attractive to outsiders, thus drawing in greater participation from the labor market and reducing the marginal type \( \theta^* \).

Now, \( r > 0 \) implies that risk aversion enters the participation decision. Raising incentives no longer unambiguously increases participation, since higher bonuses load more risk onto the compensation, which risk averse agents dislike. In particular, if the level of risk aversion is sufficiently low, then the effect will be unambiguous as before, the marginal type will fall, and paying bonuses will attract more managers. However, if risk aversion is sufficiently high (namely that \( r > \frac{\gamma^2b\sigma^2}{rc^2} \)), then raising incentives will actually repel managers and cause the marginal manager \( \theta^* \) to rise in \( b \). This shows that the relationship between performance pay and participation is subtle in the presence of risk aversion. Importantly, risk aversion does not affect the incentive margin as directly as it does the participation margin, which is somewhat surprising given the traditional
tradeoff between risk and incentives. The relationship is more nuanced in that risk averse agents dislike risky compensation, and therefore will stay away from firms that offer large incentive packages.

To see this visually, consider the isoquants from the participation decision of the profit function. If risk aversion is sufficiently low \( r < \frac{\theta^*^2}{\sigma^2} \), the isoquants are visually identical to the risk-neutral case, and appear like those in Figure 4. In that case, low levels of risk-aversion are similar to risk-neutrality, delivering a downward sloping isoparticipation line (salary and bonus are substitutes). But if risk-aversion is sufficiently high \( r > \frac{\theta^*^2}{\sigma^2} \), then the isoprofit line actually slopes upward, as shown in Figure 5. As the firm raises bonuses, this loads risk onto the manager, repelling the lower quality managers. In order to counteract this effect and keep the participation level fixed, the firm raises salary, thereby attracting those risk-averse managers back to the firm. Thus, when managers are sufficiently risk-averse, salary and bonus are complements, rather than substitutes, and they therefore reinforce one another in obtaining participation.

Because the ex-post profit function is identical under risk-aversion and risk-neutrality, the isoprofit curve will look similar as before. In Figure 5, the slopes of the two isoquants (the isoparticipation curve and the isoprofit curve) reflect the marginal rates of substitution between salary and bonus. The equilibrium condition ensures that the optimal contract occurs where the slopes of the two isoquants are the same. This gives the tangency condition in the figure. Because the isoparticipation curve slopes upward, the optimal contract now occurs on the upward sloping portion of the isoprofit curve.

Solving for the optimal contract in closed form is even more difficult than before, since the risk aversion adds an additional complexity to the participation decision. Now, every manager, and therefore the marginal manager, requires a risk premium to guarantee participation. This risk premium depends on the bonus \( b \), and therefore, so does the participation decision. Because \( \theta^* \) is no longer monotonic in \( b \), the lower bound of integration in expected profit \( \Pi(s,b) \) makes the first order condition from the firm’s problem more complex. Nonetheless, it is still possible to arrive at an implicit solution that can deliver intuition.

**Proposition 3** *A firm contracting with a risk-averse agent will select an optimal contract \((s,b)\) that satisfies*

\[
s = \left[ \theta^*(1-b) - \frac{1 - F(\theta^*)}{f(\theta^*)} b \right] \frac{b \theta^* \gamma^2}{c} - k.
\]
The formula for the optimal bonus in Proposition 3 is more complex than in Proposition 2 because of the risk aversion, but the underlying forces at work are the same. Observe that the optimal bonus still moves in tandem with the marginal-inframarginal rate (MIR). Therefore, when distribution places a large mass on $\theta > \theta^*$, the MIR is small, and so the need for sorting is small. As such, the firm need only grant a small bonus to induce workers to accept the job. A similar argument holds in the other direction when the distribution places a small mass on $\theta > \theta^*$.

Notice the similarity here with the canonical model of a principal contracting with a risk averse agent under CARA preferences and linear contracts. In the canonical model, the optimal bonus is $\tilde{b} \equiv 1/(1 + cr\sigma^2)$. Both there and here, the optimal contract falls
in the cost of effort parameter $c$, the coefficient of absolute risk aversion $r$, and variance on output $\sigma^2$.

### 4.3 Comparative Statics

The model with risk-aversion introduces a handful of new exogenous parameters that deliver novel comparative statics.

The relationship between the optimal bonus with sorting ($b^*$) and without sorting ($\hat{b}$) is more subtle here. Before, sorting uniformly depressed the bonus coefficient relative to the benchmark without sorting. Now the presence of risk aversion introduces the reliance on $\gamma$, the quality of the match between the firm and the manager. A straightforward calculation, proved in the appendix, leads to the following result.

**Corollary 3** If $\gamma$ is sufficiently small, sorting inflates optimal PPS. If $\gamma$ is sufficiently large, sorting dampens optimal PPS if $c$, $r$, or $\sigma^2$ are sufficiently small.

This gives the condition under which the optimal bonus in the canonical model without sorting ($\hat{b}$) exceeds the optimal bonus in this model with sorting ($b^*$). The proof of Corollary 3 in the Appendix gives the threshold level $\gamma^*$ above which $\hat{b} < b^*$. Intuitively, Corollary 3 shows that the sorting effect dampens the optimal bonus when the complementarity between the firm and the manager is large and the size of the agency problem is small (the product $cr\sigma^2$ tracks the magnitude of the agency problem, as increases in any of the individual parameters $c$, $r$, or $\sigma^2$ will exacerbate the agency problem between the manager and the firm). Said differently, the sorting effect inflates the optimal bonus, relative to the canonical model, when the complementarity between the firm and the manager is small. This gives a testable prediction on when the firm will reduce incentives because of the sorting effect.

To interpret further, observe that the complementarity parameter itself affects the agent’s behavior. From his incentive constraint, the agent’s effort increases in $\gamma$ since he works harder when his marginal productivity of labor is high. Furthermore, the optimal $\theta^*$ decreases in $\gamma$ since the firm can hire lower ability managers when the quality of the match with each manager is high. Therefore, when $\gamma$ is small, the manager reduces effort and the pool of managers attracted to the firm shrinks (since $\theta^*$ rises). To compensate for this, the firm raises incentives $b^*$. When the complementarity between the firm and manager is large, the forces work in the opposite direction, with a slight adjustment.
Now, a large $\gamma$ increases effort and expands the pool of managers attracted to the firm (since $\theta^*$ sinks), causing the firm to reduce the bonus if the agency problem is small. When $c$, $r$, or $\sigma^2$ is large, leading to a large agency problem, then this will depress both $b^*$ and $\hat{b}$, but reduce $\hat{b}$ by a larger magnitude, so $\hat{b} < b^*$.

This model under risk aversion provides a theoretical link between the complementarity parameter $\gamma$ and the sorting effect of dampening PPS. Firms and industries where the quality to match is high should experience smaller incentive packages because of sorting. Finally, the next corollary shows when the negative relationship between risk and incentives from the canonical model still holds under sorting.

**Corollary 4** Under risk aversion, there is a negative relationship between risk and incentives ($\partial b / \partial \sigma < 0$) if and only if $s < \bar{u}$.

This shows that the firm’s choice of salary is critical in determining whether the classic tradeoff between risk and incentives continues to hold. Recall from earlier results in this section that the firm’s salary determines whether sorting is positive ($\theta^* > 0$), as well as whether the marginal manager $\theta^*$ rises or falls in the bonus. To see these results comprehensively, refer to Figure 3, which lays out the main comparative statics as a function of the firm’s choice of salary.

<table>
<thead>
<tr>
<th></th>
<th>$s &lt; \bar{u}$</th>
<th>$\bar{u} &lt; s &lt; \bar{u} + RP$</th>
<th>$\bar{u} + RP &lt; s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta^*$</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\partial \theta^* / \partial b$</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\partial \theta^* / \partial \sigma$</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\partial b / \partial \sigma$</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Figure 3.

To walk through this figure, first suppose that the outside option exceeds the salary, so $s < \bar{u}$. In this case, positive sorting does not impact the classic tradeoff between risk
and incentives. Even though the contract must now solve the dual problem of attracting
talent and motivating effort, a risk-averse agent still dislikes volatility in his output, and
this interferes with incentive provision. Because of the manager’s risk-aversion, the firm
reduces its incentives to prevent the manager from bearing excessive risk. As the level
of this risk ($\sigma^2$) increases, the firm reduces incentives even further. In response to this
reduction in incentives, the firm raises $\theta^*$, thereby attracting better workers.

On the other hand, if the salary exceeds the outside option ($s > \bar{u}$), this reverses the
classic tradeoff between risk and incentives. As Figure 3 illustrates, in this intermediate
range of $s$ ($\bar{u} < s < \bar{u} + \text{the risk premium}$), the firm still achieves positive sorting ($\theta^* > 0$),
but the relationship between risk and incentives is now positive, rather than negative.
As output risk ($\sigma^2$) increases, the firm raises incentives. In response to these higher
incentives, the firm also increases $\theta^*$, attracting better workers. Thus, the relationship
between $\theta^*$ and $\sigma^2$ is still positive, as before. The only remaining case is when salary
exceeds the outside option and the risk premium ($s > \bar{u} + \text{RP}$). But this case is
degenerate, since sorting is negative ($\theta^* < 0$), even though the relationship between risk
and incentives is positive.

These results on risk and incentives do not contradict the results of Dutta (2008) or
Baker and Jorgensen (2003). Those papers both find a negative relationship between
output risk and incentives as here, but a positive relationship between information risk
and incentives. Output risk is determined by $\sigma^2$, whereas information risk is determined
by the variance of $f$. Instead, here I expand on the relationship between output risk
and incentives, and show that this relationship depends partly on the choice of salary,
which itself is determined endogenously to solve the dual sorting and incentives problem.
And Proposition 3, which gives an expression for the optimal salary, shows the firm will
raise the salary when the fixed cost of hiring ($k$) falls, the degree of complementarity
with the firm ($\gamma$) rises, and the cost of effort ($c$) falls. Any of these changes in the
exogenous parameters can raise the salary above the outside option, at which point, the
relationship between risk and incentives tips to become positive. This corollary may help
explain why the negative relationship between risk and incentive does not hold under
all circumstances (Prendergast 2002).

Finally, collect this last result on the relationship between risk and the marginal
manager $\theta^*$.

**Corollary 5** As output risk ($\sigma^2$) increases, the firm hires better managers ($\theta^*$ rises).
The preceding paragraphs provided the logic behind the result, which stems from the interaction of the comparative static $\partial b/\partial \sigma$ and $\partial \theta^*/\partial b$. As Figure 3 shows, the individual comparative statics have the same sign, and therefore $\partial \theta^*/\partial \sigma > 0$ for any choice of salary. Intuitively, when output risk increases, regardless of what happens to the choice of bonus, the firm will uniformly hire better managers. Since a risk averse manager dislikes volatility in his income, an increase in output risk will raise his risk premium, thereby making the manager worse off. Only the best managers with the highest marginal impact on firm output will then accept the job.

5 Conclusion

Performance pay serves multiple goals: it provides incentives to existing managers, it attracts new talent, it retains existing managers, it aligns interests between shareholders and managers, it grants ownership to employees, and so on. The vast academic literature on performance pay has focused almost exclusively on incentive effects. This paper incorporates the dual incentive and sorting effects of performance pay, and puts forth a simple and tractable model that provides basic insight into how a firm can solve the two problems jointly. The main message is that sorting dampens Pay-Performance Sensitivity (PPS), which helps explain why empirical estimates of PPS are much lower than those predicted by theory. While a small amount of performance pay is necessary in order to sort good managers from bad, excessive performance pay simply transfers rent unnecessarily to the agent, and so including a sorting effect dampens the level of the bonus.

The corollaries in the paper provide a wealth of additional implications and empirical predictions. I provide conditions when the optimal bonus increases in “information risk,” the measure of the variation between managers. Firms will use salaries instead of bonuses to adjust for changes in its economic environment, suggesting that salaries are more nimble instruments than one might expect. And risk aversion introduces a slew of comparative statics with respect to changes in output risk. In large part, the negative relationship between risk and incentives still holds, but only if the firm sets a sufficiently low salary. And regardless of the salary, the firm will always hire better workers in response to increases in output risk.

Combining the sorting and incentive effects of performance pay is possible exactly because both problems are made simple and tractable. Research has tried for dozens
of years to solve the full-blown combined moral hazard and adverse selection problems, and this has not yet yielded strong results. This paper is part of a broader agenda to use more simple contracts in real world settings to provide some theoretical guidance. Future work in this area will expand on the sorting and incentive effects of performance pay to include repeated interactions, retention effects, subjective bonuses, and multiple managers.

## 6 Appendix A: Proofs of Propositions

**Proof of Proposition 1:**

Define the participation set for a contract \((s, b)\) to be

\[
P(s, b) = \{ \theta \in \Theta | E[w|\theta] \geq \bar{u} \}.
\]

For a given contract \((s, b)\) the expected profits of the firm is

\[
E\pi(s, b) = \int_{P(s, b)} E[\pi|\theta] f(\theta) d\theta,
\]

where \(E[\pi|\theta] = \gamma \theta (1 - b) - s - k\). Now if \(b > 1\), \(E[\pi|\theta] < 0\). Thus the firm will never set \(b > 1\). If \(b \leq 1\), \(E[\pi|\theta]\) increases in \(\theta\), and thus is monotonic in \(\theta\).

Consider a contract \((s, b) = (\bar{u}, 0)\). Observe that \(P(\bar{u}, 0) = \Theta\), so

\[
E\pi(\bar{u}, 0) = \gamma \mu_\theta - \bar{u} - k
\]

is the firm’s expected profit when it pays only a flat salary, no bonus, and attracts the entire population. Call this contract \((\bar{u}, 0)\) the fallback contract.

Suppose \(b < 0\). Then either \(s < \bar{u}\) or \(s \geq \bar{u}\). If \(s < \bar{u}\), then \(P(s, b) = \emptyset \subset \Theta = P(\bar{u}, 0)\). By monotonicity of expected profit, \(E\pi(s, b) < E\pi(s, 0)\), and thus the fallback contract generates more profit for the firm. Alternatively if \(s \geq \bar{u}\), \(P(s, b) = [0, \theta^*] \subset \Theta = P(s, 0)\). By monotonicity of expected profit, \(E\pi(s, b) < E\pi(s, 0)\) and the fallback contract again dominates. In either case, the firm will never select \(b < 0\), since it can always do better with the fallback contract.

Now suppose \(b \geq 0\). If \(s > \bar{u}\), then \(P(s, b) = \Theta\). But the firm could always do better by lowering the salary to \(\bar{u}\), which not affect participation, but increases profits. Formally, \(P(\bar{u}, b) = \Theta = P(s, b)\) and \(\gamma \theta^*(1 - b) - \bar{u} - k > \gamma \theta(1 - b) - s - k\). Then
\[ E\pi(\bar{u}, b) = \gamma \mu_0(1 - b) - \bar{u} - k > \gamma \mu_0(1 - b) - s - k = E\pi(s, b). \]

Thus, \( s \leq \bar{u} \). Therefore, the feasibility set is \( FS \equiv \{(s, b) | s \leq \bar{u}, b \geq 0\} \). Define the efficient set to be all contracts that implement efficient sorting:

\[ ES \equiv \{(s, b) | \theta^* = \theta^{FB}, s \leq \bar{u}, b \geq 0\} \subset FS. \]

Observe that every contract in the efficient set generates expected surplus less expected wage payments. Because every such contract implements \( \theta^{FB} \), the contract parameters do not change the variable of integration above, and only lower the expected wage payment. Thus maximizing expected surplus over the efficient set is equivalent to minimizing the expected wage payments:

\[ \arg \max_{ES} E\pi(s, b) = \arg \min_{ES} \int_{\theta^{FB}} E[w|\theta] f(\theta) d\theta. \]

Now for every contract in the efficient set, \( s = \bar{u} - \gamma b^{FB} \) and \( b \geq 0 \). Therefore,

\[ E[w|\theta] = s + b\gamma \theta = \bar{u} + b\gamma (\theta - \theta^{FB}) \]

is a linear and nondecreasing function in \( b \) for all \( \theta > \theta^{FB} \). Minimizing this function is equivalent to setting \( b \) as low as possible.

Let \( \eta > 0 \). Take \( b = \eta \) and \( s = \bar{u} - k\eta \). Observe that \( \theta^*(s, b) = \theta^{FB} \). And

\[ E[w|\theta] = s + b\gamma \theta = \bar{u} + \eta(\gamma \theta - k). \]

Now

\[ \int_{\theta^{FB}} E[w|\theta] f(\theta) d\theta = \int_{\theta^{FB}} \bar{u} f(\theta) d\theta + \eta \int_{\theta^{FB}} (\gamma \theta - k) f(\theta) d\theta \]

And therefore,

\[ E\pi(s, b) = E[TS](1 - \eta) - \int_{\theta^{FB}} \bar{u} f(\theta) d\theta. \]

The fallback contract generates expected profits
\[ E\pi(\bar{u}, 0) = \int_{\Theta} (\gamma \theta - \bar{u} - k) f(\theta) d\theta \]
\[ = \int_{0}^{\theta_{FB}} (\gamma \theta - k - \bar{u}) f(\theta) d\theta + \int_{\theta_{FB}}^{\infty} (\gamma \theta - \bar{u} - k) f(\theta) d\theta \]
\[ = \int_{0}^{\theta_{FB}} (\gamma \theta - k - \bar{u}) f(\theta) d\theta + E[TS] - \int_{\theta_{FB}}^{\infty} \bar{u} f(\theta) d\theta \]

Thus, \( E\pi(s, b) > E\pi(s, 0) \) if and only if

\[ \eta < \frac{\int_{0}^{\theta_{FB}} (\bar{u} - (\gamma \theta - k)) f(\theta) d\theta}{E[TS]} \]

The denominator is positive because expected total surplus is positive. The numerator is also positive because \( \gamma \theta < k \) for each \( \theta < \theta_{FB} \). Therefore,

\[ \lim_{\eta \downarrow 0} E\pi(s, b) = E[TS] - \int_{\theta_{FB}}^{\infty} \bar{u} f(\theta) d\theta > E\pi(\bar{u}, 0). \]

Thus the optimal contract is \( s = \bar{u} - k\eta \rightarrow \bar{u} \) and \( b = \eta \rightarrow 0 \). Thus \( s \approx \bar{u} \) and \( b \approx 0 \).

\[ \blacksquare \]

Proof of Proposition 2:

The firm chooses salary \( s \) and bonus \( b \) to solve

\[ \max_{s, b} \int_{\theta^{*}(s, b)}^{\infty} E[\pi|\theta] f(\theta) d\theta \]

Using Leibnitz’s Rule, the first order conditions are

\[ E[\pi|\theta^{*}] f(\theta^{*}) \frac{\partial \theta^{*}}{\partial s} = \int_{\theta^{*}}^{\infty} \frac{\partial E[\pi|\theta]}{\partial s} f(\theta) d\theta \]
\[ E[\pi|\theta^{*}] f(\theta^{*}) \frac{\partial \theta^{*}}{\partial b} = \int_{\theta^{*}}^{\infty} \frac{\partial E[\pi|\theta]}{\partial b} f(\theta) d\theta \]

Combining these two equations leads to the equilibrium condition

\[ \frac{\partial \theta^{*}}{\partial b} \int_{\theta^{*}}^{\infty} \frac{\partial E[\pi|\theta]}{\partial s} f(\theta) d\theta = \frac{\partial \theta^{*}}{\partial s} \int_{\theta^{*}}^{\infty} \frac{\partial E[\pi|\theta]}{\partial b} f(\theta) d\theta. \]

(30)
Now, the ex-post expected profit \( E[\pi|\theta] = E[x|\theta] - E[w|\theta] - k = \gamma \theta e(1 - b) - s - k. \) Plugging in the incentive constraint \( e = b \gamma \theta / c, \)

\[
E[\pi|\theta] = \frac{(\gamma \theta)^2}{c} b(1 - b) - s - k.
\]

(31)

The derivatives of expected profit will respect to salary and bonus are, respectively:

\[
\frac{\partial E[\pi|\theta]}{\partial s} = -1 \quad \text{and} \quad \frac{\partial E[\pi|\theta]}{\partial b} = \frac{(\gamma \theta)^2}{c} (1 - 2b)
\]

(32)

Recall that \( \theta^* = \sqrt{2c(\bar{u} - s) / b \gamma}. \) The partial derivatives are:

\[
\frac{\partial \theta^*}{\partial s} = \frac{-c}{(b \gamma)^2 \theta^*} \quad \text{and} \quad \frac{\partial \theta^*}{\partial b} = -\frac{\theta^*}{b}
\]

(33)

Combining these gives

\[
\frac{\partial \theta^*}{\partial b} \frac{\partial \theta^*}{\partial s} = \frac{\partial \theta^*}{\partial s} \frac{b \theta^* \gamma^2}{c}
\]

(34)

Combining (5) with the equilibrium condition (3) gives

\[
-\frac{\partial \theta^*}{\partial b} (1 - F(\theta^*)) = \frac{\partial \theta^*}{\partial s} \int_{\theta^*}^{\infty} \frac{(\gamma \theta)^2}{c} (1 - 2b) f(\theta) d\theta.
\]

Combining with (7) gives

\[
\left( -\frac{b \theta^* \gamma^2}{c} \right) (1 - F(\theta^*)) = \int_{\theta^*}^{\infty} \frac{(\gamma \theta)^2}{c} (1 - 2b) f(\theta) d\theta.
\]

Rearranging and simplifying,

\[
b = \left( 2 - \frac{\theta^* \gamma^2}{E[\theta^2|\theta > \theta^*]} \right)^{-1}.
\]

Inserting (5) into the first order condition for \( s \) gives

\[
E[\pi|\theta^*] f(\theta^*) \left( -\frac{\partial \theta^*}{\partial s} \right) = 1 - F(\theta^*).
\]

Combining with (4), (6), and simplifying, this yields

\[
s = \left[ \theta^*(1 - b) - \frac{1 - F(\theta^*)}{f(\theta^*)} b \right] \frac{b \gamma^2 \theta^*}{c} - k.
\]

\[\blacksquare\]
Proof of Corollary 1:

Let \( f(\theta) = \frac{1}{2a} \) for all \( \theta \in \Theta = [\mu - a, \mu + a] \) for some \( \mu > 0 \) and \( \mu > a > 0 \). Simple calculation of uniform densities shows \( E\theta = \mu \) and \( V\theta = a^2/3 \). Now

\[
Pr(\theta > \theta^*) = \frac{\mu + a - \theta^*}{2a}
\]

And \( f(\theta|\theta > \theta^*) = f(\theta)/Pr(\theta > \theta^*) = (\mu + a - \theta^*)^{-1} \). So

\[
E \equiv E[\theta^2|\theta > \theta^*] = \int_{\theta^*}^{\mu + a} \theta^2 f(\theta|\theta > \theta^*)d\theta
\]

where \( \theta^* > \mu - a \). Calculating this integral,

\[
E = \frac{(\mu + a)^3 - \theta^*^3}{3(\mu + a - \theta^*)} \tag{C1}
\]

Observe that \( \theta^* < \mu + a \), so \( E > 0 \). Let \( X = 2c(\bar{u} - s) \). Then \( \theta^* = \sqrt{X/b}\gamma \). From Proposition 2, \( b = (2 - \theta^*^2/E)^{-1} \). Combining with (C1),

\[
b^{-1} = 2 - \frac{3(\mu + a - \theta^*)\theta^*^2}{(\mu + a)^3 - \theta^*^3}
\]

Rearranging,

\[
3(\mu + a - \theta^*)\theta^*^2 = [(\mu + a)^3 - \theta^*^3](2 - b^{-1})
\]

Substituting in \( \theta^* = \sqrt{X/b}\gamma \),

\[
3 \left( \mu + a - \frac{\sqrt{X}}{b\gamma} \right) \frac{X}{b^2\gamma^2} = \left[ (\mu + a)^3 - \frac{X^{3/2}}{b^3\gamma^3} \right] (2 - b^{-1})
\]

Rearranging,

\[
0 = \frac{5}{3} b^4 \gamma^4 (\mu + a)^3 - 3 b^2 X \gamma^2 (\mu + a) + b X^{3/2} \gamma + X^{3/2} \gamma
\]

which is a fourth order polynomial in \( b \). Using the Implicit Function Theorem, differentiate with respect to \( a \):

\[
0 = \frac{5}{3} \gamma^4 \left( b^4 (\mu + a)^2 + 4 b^3 \frac{\partial b}{\partial a} \right) - 3 X \gamma^2 \left( b^2 + (\mu + a) 2b \frac{\partial b}{\partial a} \right) + X^{3/2} \gamma \frac{\partial b}{\partial a}
\]

Solving for \( \frac{\partial b}{\partial a} \),

38
\[
\frac{\partial b}{\partial a} = \frac{5\gamma^4 b^4 (\mu + a)^2 - 3X \gamma^2 b^2}{\frac{20}{3} b^3 \gamma^4 + 6b X \gamma^2 - X^{3/2} \gamma}.
\]

Now \(\frac{\partial b}{\partial a} > 0\) if

\[
5\gamma^2 b^2 (\mu + a)^2 > 3X \quad \text{and} \quad 6X \gamma^2 b > \frac{20}{3} b^3 \gamma^4 + X^{3/2} \gamma
\]

or \(5\gamma^2 b^2 (\mu + a)^2 < 3X \quad \text{and} \quad 6X \gamma^2 b < \frac{20}{3} b^3 \gamma^4 + X^{3/2} \gamma.
\]

Clearly there exist \(\gamma, \mu, a, X\) such that either condition holds. In particular, the second condition holds if

\[
X > \frac{5}{3} \gamma^2 b^2 (\mu + a)^2 \quad \text{and} \quad \frac{10b^2 \gamma^2}{9} + \frac{\sqrt{X}}{6b\gamma} > 1
\]

both which occur if \(c\) or \(\bar{u}\) (and hence \(X\)) are large.

\section*{Proof of Proposition 3:}

The firm chooses salary \(s\) and bonus \(b\) to solve

\[
\max_{s,b} \int_{\theta^*(s,b)}^{\infty} E[\pi|\theta] f(\theta) d\theta
\]

Using Leibnitz’s Rule, the first order conditions are

\[
E[\pi|\theta^*] f(\theta^*) \frac{\partial \theta^*}{\partial s} = \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial s} f(\theta) d\theta
\]

\[
E[\pi|\theta^*] f(\theta^*) \frac{\partial \theta^*}{\partial b} = \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial b} f(\theta) d\theta
\]

Combining these two equations leads to the equilibrium condition

\[
\frac{\partial \theta^*}{\partial b} \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial s} f(\theta) d\theta = \frac{\partial \theta^*}{\partial s} \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial b} f(\theta) d\theta
\] \hspace{1cm} (35)

Now, \(E[\pi|\theta] = E[x|\theta] - E[w|\theta] = \gamma \theta e (1 - b) - s\). Plugging in (IC),

\[
E[\pi|\theta] = \frac{\gamma^2 \theta^2 b}{c} (1 - b) - s - k
\] \hspace{1cm} (36)
The derivatives of expected profit will respect to salary and bonus are, respectively:

\[
\frac{\partial E[\pi|\theta]}{\partial s} = -1 \quad \text{and} \quad \frac{\partial E[\pi|\theta]}{\partial b} = \frac{\gamma^2 \theta^2}{c} \left(1 - 2b\right) \tag{37}
\]

Let \( X = 2c(\bar{u} - s) + crb^2\sigma^2 \). Observe that

\[
\theta^* = \frac{\sqrt{X}}{b\gamma}. \tag{38}
\]

The partial derivatives of \( X \) are

\[
\frac{\partial X}{\partial s} = -2c \quad \text{and} \quad \frac{\partial X}{\partial b} = 2crb\sigma^2 = -\frac{\partial X}{\partial s} \cdot rb\sigma^2. \tag{39}
\]

Differentiating (38) gives

\[
\frac{\partial \theta^*}{\partial s} = \frac{1}{2b\gamma \sqrt{X}} \left( \frac{\partial X}{\partial s} \right) \tag{40}
\]

\[
\frac{\partial \theta^*}{\partial b} = \frac{1}{2b\gamma \sqrt{X}} \left( \frac{\partial X}{\partial b} \right) - \frac{\sqrt{X}}{b^2\gamma} \tag{41}
\]

Combining (41) with (39) gives

\[
\frac{\partial \theta^*}{\partial b} = \frac{1}{2b\gamma \sqrt{X}} \left( \frac{\partial X}{\partial s} \right) \left(-rb\sigma^2\right) - \frac{\sqrt{X}}{b^2\gamma}. \]

Substituting in (40) and (38),

\[
\frac{\partial \theta^*}{\partial b} = -rb\sigma^2 \frac{\partial \theta^*}{\partial s} - \frac{\theta^*}{b} \tag{42}
\]

Now, combining (38), (39), (40) gives

\[
\frac{\partial \theta^*}{\partial s} = \frac{c}{2b\gamma \sqrt{X}} = \frac{c}{(b\gamma)^2 \theta^*} \tag{43}
\]

Combining (37) with the equilibrium condition (35) gives

\[
- \frac{\partial \theta^*}{\partial b} \left(1 - F(\theta^*)\right) = \frac{\partial \theta^*}{\partial s} \int_{\theta^*}^{\infty} \frac{\gamma^2 \theta^2}{c} (1 - 2b)f(\theta)d\theta.
\]

Combining this with (42) gives

\[
\left( r\sigma^2b \frac{\partial \theta^*}{\partial s} + \frac{\theta^*}{b} \right) \cdot \left(1 - F(\theta^*)\right) = \left( \frac{\partial \theta^*}{\partial s} \right) \int_{\theta^*}^{\infty} \frac{\gamma^2 \theta^2}{c} (1 - 2b)f(\theta)d\theta.
\]
Collecting terms,
\[ r b \sigma^2 \frac{\partial \theta^*}{\partial s} + \frac{\theta^*}{b} = \frac{\partial \theta^*}{\partial s} \frac{\gamma^2 (1 - 2b)}{c} E \]

Where \( E = E[\theta^2|\theta > \theta^*] \). Substituting in (43),
\[ \frac{\theta^*}{b} = \frac{c}{(b \gamma)^2 \theta^*} \left( r b \sigma^2 - \frac{\gamma^2 (1 - 2b)}{c} E \right) \]

Rearranging and simplifying,
\[ b = \left( 2 + \frac{cr \sigma^2 - \theta^* \gamma^2}{\gamma^2 E} \right)^{-1} \]

Inserting (37) into the first order condition for \( s \) gives
\[ E[\pi|\theta^*]f(\theta^*)\left( -\frac{\partial \theta^*}{\partial s} \right) = 1 - F(\theta^*). \]

Combining with (36) and (43), and solving for \( s \),
\[ s = \left[ \theta^*(1 - b) - \frac{1 - F(\theta^*)}{f(\theta^*)} b \theta^* \gamma^2 \right] \frac{b \theta^* \gamma^2}{c} - k. \]

\[ \square \]

\textbf{Proof of Corollary 3:}

Let \( \hat{b} = (1 + cr \sigma^2)^{-1} \) be the bonus from the canonical model, detailed in Appendix B.

From Proposition 3,
\[ b^* = \left( 2 + \frac{cr \sigma^2 - (\gamma \theta^*)^2}{\gamma^2 E} \right)^{-1} \]

where \( E = E[\theta^2|\theta > \theta^*] \). Now \( b^* < \hat{b} \) iff
\[ \gamma^2 E (1 + cr \sigma^2) < 2 \gamma^2 E + cr \sigma^2 - (\gamma \theta^*)^2. \]

Collecting terms, this holds iff
\[ cr \sigma^2 < \frac{\gamma^2 (E - \theta^*^2)}{\gamma^2 E - 1} \equiv W \]
Now $\theta^* < E$, so $W > 0$ iff $E > 1/\gamma^2$. Let $\gamma^* = 1/\sqrt{E}$. For $\gamma < \gamma^*$, $W < 0$ and so $b^* > \hat{b}$. For $\gamma > \gamma^*$, $b^* < \hat{b}$ if $cr\sigma^2 < W$. ■

Proof of Corollary 4:

$\theta^*$ in this model is

$$\theta^* = \frac{\sqrt{2c(\bar{u} - s) + crb^2\sigma^2}}{b\gamma}$$  \hspace{1cm} (44)

Now after some algebra,

$$\frac{\partial \theta^*}{\partial \sigma} = \frac{cr\sigma}{\theta^* \gamma^2} > 0.$$  \hspace{1cm} (45)

Let $E \equiv E[\theta^2|\theta > \theta^*] = (\int_{\theta^*}^{\infty} \theta^2 f(\theta) d\theta) / (1 - F(\theta^*))$.

After simplifying,

$$\frac{\partial E}{\partial \sigma} = \frac{f(\theta^*)}{1 - F(\theta^*)} \frac{\partial \theta^*}{\partial \sigma} \left( E[\theta^2|\theta > \theta^*] - \theta^* \gamma^2 \right) > 0,$$  \hspace{1cm} (46)

since $f$ and $F > 0$, $\frac{\partial \theta^*}{\partial \sigma} > 0$ from (45), and $\theta^* < E[\theta^2|\theta > \theta^*]$. Let

$$W = (cr\sigma^2 - (\gamma \theta^*)^2)/\gamma^2 E.$$

From Proposition 3, $b = (2 + W)^{-1}$. Clearly $\frac{\partial b}{\partial \sigma} < 0$ iff $\frac{\partial W}{\partial \sigma} > 0$. It remains to show $W' \equiv \frac{\partial W}{\partial \sigma} > 0$. Now,

$$W' = \frac{\gamma^2 E \left( 2cr\sigma - 2\gamma^2 \theta^* \frac{\partial \theta^*}{\partial \sigma} \right) - (cr\sigma^2 - \gamma^2 \theta^2) \gamma^2 \frac{\partial E}{\partial \sigma}}{(\gamma^2 E)^2}.$$  

This term is positive if

$$\gamma^2 E \left( 2cr\sigma - 2\gamma^2 \theta^* \frac{\partial \theta^*}{\partial \sigma} \right) > (cr\sigma^2 - \gamma^2 \theta^2) \gamma^2 \frac{\partial E}{\partial \sigma}.$$  

From (45), the left-hand side is zero. From (46), $\frac{\partial E}{\partial \sigma} > 0$. So $W' > 0$ iff

$$cr\sigma^2 < \gamma^2 \theta^*.$$  

Plugging in (44), this occurs iff $s < \bar{u}$. So $W' > 0$ and thus $\frac{\partial b}{\partial \sigma} < 0$. ■
Appendix B: The Canonical Agency Model

Here is the canonical agency model of a risk-neutral principal contracting with an agent.

The agent exerts effort \( e \) at cost \( C(e) = 0.5ce^2 \). Firm output is \( x = e + \epsilon \), where \( \epsilon \) follows a distribution \( g \) with mean 0 and variance \( \sigma^2 \). The firm pays wages \( w = s + bx \). First best maximizes \( e - C(e) \), yielding \( e^{FB} = 1/c \).

7.1 Risk Neutral Agent

Assume the agent is risk neutral. He therefore maximizes \( E[w] - C(e) \), yielding the incentive constraint (IC) \( \hat{e} = b/c \). In equilibrium, the cost of effort is \( C(\hat{e}) = b^2/(2c) \). The agent accepts the contract if \( E[w] - C(e) \geq \bar{u} \). The principal will set \( s \) such that this participation constraint binds. Plugging in the incentive constraint and solving for \( s \), this gives \( s = \bar{u} - b^2/(2c) \).

The principal’s payoff is \( E[\pi] = E[x] - E[w] = \hat{e}(1 - b) - s \). The principal can implement first best by selecting \( b = 1 \) and this gives a salary \( s = \bar{u} - 1/2c \). The agent’s payoff is \( \bar{u} \) and the principal earns \( E[\pi] = -s = 1/2c - \bar{u} \).

7.2 Risk Averse Agent

Assume the agent is risk averse with CARA preferences. Therefore, the agent maximizes the certainty equivalent of his compensation, which is

\[
CE = s + be - c(e) - \frac{r}{2}b^2\sigma^2
\]

where \( r \) is the coefficient of absolute risk aversion. The incentive constraint remains \( \hat{e} = b/c \) but the binding participation constraint includes the risk premium, so \( s = \bar{u} - b^2/(2c) + \frac{r}{2}b^2\sigma^2 \).

The firm maximizes \( E[\pi] = E[x] - E[w] = \hat{e}(1 - b) - s \). Plugging in (IC) and the binding participation constraint, this becomes

\[
\max_b \frac{b}{c} - \frac{b^2}{2c} - \frac{r}{2}b^2\sigma^2
\]

The solution to this is \( \hat{b} = (1 + rc\sigma^2)^{-1} \).
References


