1) Calculate the continuously compounded zero-coupon rates that correspond to the market quotes, $r_T$.

\[ z_T = \ln \left(1 + r_T \times T \right) / T \]

<table>
<thead>
<tr>
<th>Maturities</th>
<th>3 month</th>
<th>6 month</th>
<th>1 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of year, $T$</td>
<td>0.25</td>
<td>0.5</td>
<td>1.00</td>
</tr>
<tr>
<td>Market Rate, $r_T$</td>
<td>2.00%</td>
<td>2.50%</td>
<td>2.50%</td>
</tr>
<tr>
<td>Cont. Compounded Rate, $z_T$</td>
<td>1.99502%</td>
<td>2.4845%</td>
<td>2.46926%</td>
</tr>
</tbody>
</table>

2) If both $R$ and $Z = 2.5\%$, what can you say about $Y$ in the following, and why?

\[
\begin{bmatrix}
1 \\
1 + R/2
\end{bmatrix}
\begin{bmatrix}
1 \\
1 + Y/2
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
1 + Z
\end{bmatrix}
\text{invert to } (1 + R/2)(1 + Y/2) = (1 + Z) \Rightarrow
\]

\[ (1 + Y/2) = \frac{(1 + Z)}{(1 + R/2)} \Rightarrow Y = 2 \times \left[ \frac{(1 + Z)}{(1 + R/2)} - 1 \right] = 2.4691\%, \text{ and } < 2.5\%, \text{ as it must be because } 2.5\% \text{ compounded two times is greater than } 2.5\% \text{ uncompounded.} \]

3) If continuously-compounded rate, $z$, is 2.46926\%, then what is duration value of the bond? The discrete or analytic duration value is fine. For the analytic value, $\frac{dB}{dz}/B$ is the duration formula, and the derivative follows from $\frac{de^{xt}}{dx} = te^{xt}$. Please show your work.

\[
- \frac{dB}{dz}/B = - \frac{d(e^{-z})}{dz} / e^{-z} = - \frac{-e^{-z}}{e^{z}} = 1. \text{ Duration is } T, \text{ maturity, for a zero coupon bond with a continuously compounded interest rate. For numerical approximation, the bond value, } B^0, \text{ is } 0.97561, \text{ and increasing rates } 1 \text{ bp to is } 2.46936\%, \text{ yields a } B^+ = 0.97551 \text{ value. Duration is } -(B^+-B^0)/dz/ B^0 = -(0.975561/0.0001/0.97561) = 0.99995. \]