**Quiz 1 Suggested Answers**

Please use the following rates in all questions below:

<table>
<thead>
<tr>
<th>Maturities</th>
<th>3 month</th>
<th>6 month</th>
<th>1 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of year, T</td>
<td>0.25</td>
<td>0.5</td>
<td>1.00</td>
</tr>
<tr>
<td>Market Rate, R_T</td>
<td>10.0%</td>
<td>10.0%</td>
<td>12.0%</td>
</tr>
<tr>
<td>Cont. Compounded Rate, Z_T</td>
<td>9.87705%</td>
<td>9.75803%</td>
<td>11.3329%</td>
</tr>
</tbody>
</table>

\[ z_T = \ln \left(1 + R_T \cdot T\right) / T \]

1) Please circle the correct symbol, >, = or <:

\[
\left(1 + 0.1/2\right)^2 < \left(1 + 0.1/4\right)^4,
\]

The left-hand side denominator is smaller. So >.

Or by numbers, \(4.53515 > 4.52975\)

2) If the money market is working, then \(e^{-z_T \cdot T} = \frac{1}{1 + \frac{R_T}{T}}\). Solve for the continuously compounded rate, \(z_T\), and be sure to show your work! Invert, \(e^{z_T \cdot T} = 1 + \frac{R_T}{T}\) Take logs, \(z_T \cdot T = \ln \left(1 + \frac{R_T}{T}\right)\), Divide by \(T\), \(z_T = \ln \left(1 + \frac{R_T}{T}\right) / T\). Given \(R_1 = 12.0\%\), what is \(z_1\)?

\(z_1 = \ln \left(1 + 0.12 \cdot 1\right) / 1 = 0.113329\)

3) Solve the following integral:

\[ \int_0^1 (1 + y)e^{-\int_0^y z \, dt} \, dy \]. Please state any link to finance that comes to mind.

\[ e^{-\int_0^y z \, dt} = e^{-z} \]. So, we solve \(e^{-z} \left( \int_0^1 (1 + y) \, dy \right) = e^{-z} \left( \int_0^1 dy + \int_0^1 y \, dy \right)\).

As \(y_0^1 = \int_0^1 dy\) and \(\frac{y^2}{2} \bigg|_{0}^{1} = \int_0^1 y \, dy\), we have \(e^{-z} \left( \int_0^1 + \frac{1}{2} \right) - 0 = 3e^{-z} / 2\). @ \(z_1 = 0.113329\), = 1.3929

A cash flow of \$1 has a random component that is uniformly distributed between 0 and \$1. The expected cash flow is discounted on a continuously compounded basis at rate \(z\). Importantly, the random part of the cash flow and the discount rate are unrelated and separable. The expected value is 3/2 discounted at the one-year continuously compounded zero interest rate.

To approximate the integrals with sums, we will set the interval \(dt = dy = 0.01\). Since both integral bounds of integration are zero and one, we will have 100 intervals.

\[ \int_0^1 z \, dt \approx z \sum_{i=1}^{100} 0.01 = z \cdot 1.0 \]

\[ \int_0^1 y \, dy \approx \sum_{i=1}^{100} \frac{i}{100} = 0.01, \sum_{i=1}^{100} \frac{i}{10000} = \frac{1}{10000} \sum_{i=1}^{100} i = \frac{100(100+1)/2}{10000} = 0.505 \]

\[ e^{-z} \left( \int_0^1 dy + \int_0^1 y \, dy \right) \approx e^{-z} \sum_{i=1}^{100} 0.01 \left( \sum_{i=1}^{100} 0.01 + \sum_{i=1}^{100} \frac{i}{100} \right) = e^{-z} \left( 1 + 0.505 \right) = 1.505 \, e^{-z}\]

@ \(z_1 = 0.113329\), = 1.34375