A Generalized Measure of Riskiness

Turan G. Bali\textsuperscript{a}\textsuperscript{*}
\hspace{1cm} \textsuperscript{a}McDonough School of Business, Georgetown University, Washington, D.C.20057

Nusret Cakici\textsuperscript{b}\textsuperscript{†}
\hspace{1cm} \textsuperscript{b}Graduate School of Business, Fordham University, New York, NY 10023, USA

Fousseni Chabi-Yo\textsuperscript{c}\textsuperscript{‡}
\hspace{1cm} \textsuperscript{c}Fisher College of Business, Ohio State University, Columbus, OH 43210-1144, USA

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Online Appendix

Section I provides our derivations for the generalized measure of riskiness in Appendix A, B, and C. Section II describes our estimation methodology and empirical results on the risk aversion parameter in Merton’s framework. Section III presents results from an alternative measure of physical riskiness and the corresponding measure of riskiness premium.

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\textsuperscript{*}Tel.: +1-646-312-3506; fax: +1-646-312-3451. E-mail address: turan.bali@baruch.cuny.edu

\textsuperscript{†}Tel.: +1-212 636 6776; fax: +1-212-586-0575. E-mail address: cakici@fordham.edu

\textsuperscript{‡}Tel.: +1-614-292-8477; fax: +1-614-292-7062. E-mail address: chabi-yo_1@fisher.osu.edu
I. Appendix

Appendix A

The following two lemmas are necessary to derive a formal proof of Theorem 1.

**Lemma 1.**
Let
\[
\phi[\delta](\lambda) = \begin{cases} 
\frac{\sum_{i=1}^{m} p_i (1+\lambda x_i)^{\delta-1}}{\delta} & \text{if } \delta \neq 0 \\
\sum_{i=1}^{m} p_i \log (1+\lambda x_i) & \text{if } \delta = 0 
\end{cases}
\]

There exists a unique value \(\lambda^*\) with \(0 \leq \lambda^* \leq \frac{1}{L}\) such that \(\phi[\delta](\lambda^*) = 0\). We denote \(R[\delta][g_{t+1}] = \frac{1}{\lambda^*}\).

**Lemma 2.**
The riskiness measure \(R[\delta][g_{t+1}]\) is homogeneous of degree one.

**Proof of Lemma 1.** Foster and Hart (2009) provide the proof when \(\delta = 0\). We need to prove Theorem 1 for \(\delta \neq 0\). For \(0 \leq \lambda \leq \frac{1}{L}\), we show that:

\[
\phi^{(\delta)}[0] = 0; \\
\lim_{\lambda \to \left(\frac{1}{L}\right)} \phi^{(\delta)}[\lambda] = -\infty \text{ if } \delta < 0; \\
\phi^{(\delta)'}[\lambda] = \sum_{i=1}^{m} p_i x_i (1+\lambda x_i)^{\delta-1}; \\
\phi^{(\delta)'}[0] = \sum_{i=1}^{m} p_i x_i = E(g_{t+1}) > 0; \\
\phi^{(\delta)''}[\lambda] = (\delta - 1) \sum_{i=1}^{m} p_i x_i^2 (1+\lambda x_i)^{\delta-2}.
\]

The function \(\phi^{(\delta)}\) is a strictly concave function that starts at \(\phi^{(\delta)}[0] = 0\) with a positive slope \(\phi^{(\delta)'}[0]\) and goes \(-\infty\) if \(\delta < 0\). Therefore, there exists a unique value \(\lambda^*\) such that \(\phi^{(\delta)}[\lambda^*] = 0\). We denote \(R[\delta][g_{t+1}] = \frac{1}{\lambda^*}\).
Proof of Lemma 2. It is straightforward to show that $R_\delta [g_{t+1}]$ is homogeneous of degree one:

$$R_\delta [\lambda g_{t+1}] = \lambda R_\delta [g_{t+1}].$$  \hfill (A2)

From Lemma 1, it follows that

$$E_t \left( \frac{1 + \frac{1}{R_\delta [g_{t+1}]} g_{t+1}}{\delta} \right)^\delta - 1 = E_t \left( \frac{1 + \frac{1}{R_\delta [g_{t+1}]} (\lambda g_{t+1})}{\delta} \right)^\delta - 1 = E_t \left( \frac{1 + \frac{1}{R_\delta [\lambda g_{t+1}]} (\lambda g_{t+1})}{\delta} \right)^\delta - 1 = 0.$$  \hfill (A3)

Now, we use Lemma 1 and Lemma 2 to derive a formal proof of Theorem 1.

Proof of Theorem 1. If a strategy $s$ satisfies (11), then $s$, guaranties no bankruptcy. We define

$$Y_t := \frac{1}{\delta} \left( W_{t+1}^\delta - W_t^\delta \right).$$  \hfill (A4)

and $d_t$ the decision made at time $t$. The history before the decision to accept or reject the gamble is denoted

$$f_{t-1} = (W_1, g_1, d_1, ..., W_{t-1}, g_{t-1}, d_t, g_{t+1}).$$  \hfill (A5)

If the gamble is accepted $W_{t+1} = W_t + g_{t+1}$ and $Y_t = \frac{1}{\delta} (W_t + g_{t+1})^\delta - W_t^\delta > 0$. If the gamble is rejected, $Y_t = 0$. If the gamble is accepted, we have

$$E_t (Y_t | f_{t-1}) = W_t^\delta \left( \frac{1 + \frac{g_{t+1}}{W_t}}{\delta} \right)^\delta - 1 \geq 0,$$  \hfill (A6)

and

$$E_t (Y_t | f_{t-1}) = 0.$$  \hfill (A7)

If the gamble is rejected. Notice that from lemma 1,

$$E_t \left( \frac{1 + \frac{1}{R_\delta [g_{t+1}]} (g_{t+1})}{\delta} \right)^\delta - 1 = 0.$$  \hfill (A8)
Thus, when the gamble is accepted, it follows that \( \frac{1}{W_t} \leq \frac{1}{R_\delta [g_{t+1}]} \). Hence,

\[
\frac{M [g_{t+1}]}{W_t} \leq \frac{M [g_{t+1}]}{R_\delta [g_{t+1}]}. \tag{A9}
\]

Since

\[
\frac{g_{t+1}}{W_t} \leq \frac{M [g_{t+1}]}{W_t}, \tag{A10}
\]

we combine (A9) and (A10), and have

\[
1 + \frac{g_{t+1}}{W_t} \leq 1 + \frac{M [g_{t+1}]}{R_\delta [g_{t+1}]} < \infty. \tag{A11}
\]

Also, notice that \( g_{t+1} \geq -L [g_{t+1}] \). Therefore,

\[
1 + \frac{g_{t+1}}{W_t} \geq 1 + \frac{-L [g_{t+1}]}{W_t} \geq 1 + \frac{-L [g_{t+1}]}{R_\delta [g_{t+1}]]. \tag{A12}
\]

We use (A11) and show that:

\[
Y_t = W_t^\delta \left( \frac{1 + \frac{g_{t+1}}{W_t}}{\delta} - 1 \right) \leq W_t^\delta \sup_{g_{t+1} \in G_0} \left( \frac{1 + \frac{M [g_{t+1}]}{R_\delta [g_{t+1}]]}}{\delta} - 1 \right). \tag{A13}
\]

Similarly, it can be shown that

\[
Y_t \geq W_t^\delta \min_{g_{t+1} \in G_0} \left( \frac{1 - \frac{L [g_{t+1}]}{R\delta [g_{t+1}]]}}{\delta} - 1 \right). \tag{A14}
\]

Therefore, for \( \delta < 0 \), the random variable \( Y_t \) is uniformly bounded. Now, we define

\[
X_T := \sum_{t=1}^{T} (Y_t - E (Y_t | f_{t-1})). \tag{A15}
\]

The random variable \( X_T \) is a martingale with bounded increments. Since \( E (Y_t | f_{t-1}) > 0 \),

\[
X_T := \sum_{t=1}^{T} Y_t - \sum_{t=1}^{T} E (Y_t | f_{t-1}) \leq \sum_{t=1}^{T} Y_t = \frac{1}{\delta} \left( W_{T+1}^\delta - W_1^\delta \right). \tag{A16}
\]

Bankruptcy occurs when \( W_{T+1} \to 0 \). Assume that \( \delta < 0 \), if bankruptcy occurs, \( \frac{1}{\delta} W_{T+1}^\delta \to -\infty \) and \( X_T \to \)}
Following Proposition VII-3-9 in Neveu (1975), the event $X_T \to -\infty$ has probability 0. ■

**Proof of Proposition 1.** The proofs of (i), (ii) and (iii) follow directly from Proposition 1, Lemma 1, and Lemma 2.

(iv) Consider two gambles $g, h \in \mathcal{G}$, and denote $R_\delta[g]$ and $R_\delta[h]$ the critical wealth level for the gambles $g$ and $h$ respectively. Denote

$$\lambda = \frac{R_\delta[g]}{R_\delta[g] + R_\delta[h]}.$$  \hspace{1cm} (A17)

Since the function

$$\Phi^{(\delta)}[x] = \begin{cases} \frac{(1+x)^\delta - 1}{\delta} & \text{if } \delta < 0 \\ \log(1+x) & \text{if } \delta = 0 \end{cases}$$  \hspace{1cm} (A18)

is concave, we have

$$\Phi^{(\delta)} \left[ \lambda \frac{g}{R_\delta[g]} + (1-\lambda) \frac{h}{R_\delta[h]} \right] \geq \lambda \Phi^{(\delta)} \left[ \frac{g}{R_\delta[g]} \right] + (1-\lambda) \Phi^{(\delta)} \left[ \frac{h}{R_\delta[h]} \right].$$  \hspace{1cm} (A19)

Taking the expectation of (A19) produces

$$E\Phi^{(\delta)} \left[ \lambda \frac{g}{R_\delta[g]} + (1-\lambda) \frac{h}{R_\delta[h]} \right] \geq \lambda E\Phi^{(\delta)} \left[ \frac{g}{R_\delta[g]} \right] + (1-\lambda) E\Phi^{(\delta)} \left[ \frac{h}{R_\delta[h]} \right].$$  \hspace{1cm} (A20)

Notice that

$$E\Phi^{(\delta)} \left[ \frac{g}{R_\delta[g]} \right] = 0 \text{ and } E\Phi^{(\delta)} \left[ \frac{h}{R_\delta[h]} \right] = 0.$$  \hspace{1cm} (A21)

Therefore,

$$E\Phi^{(\delta)} \left[ \frac{g+h}{R_\delta[g] + R_\delta[h]} \right] \geq 0.$$  \hspace{1cm} (A22)

From Theorem 1, it follows that

$$E\Phi^{(\delta)} \left[ \frac{g+h}{R_\delta[g] + R_\delta[h]} \right] = 0.$$  \hspace{1cm} (A23)

Hence

$$E\Phi^{(\delta)} \left[ \frac{g+h}{R_\delta[g] + R_\delta[h]} \right] \geq E\Phi^{(\delta)} \left[ \frac{g+h}{R_\delta[g+h]} \right].$$  \hspace{1cm} (A24)

Therefore,

$$R_\delta[g] + R_\delta[h] \geq R_\delta[g+h].$$  \hspace{1cm} (A25)

(v) The proof of (v) follows from (ii) and (iv).
We have
\[
E \left( 1 + \frac{\lambda + g}{R_{\delta}} \right)^{\delta} - 1 = \lambda E \left( 1 + \frac{g}{R_{\delta}} \right)^{\delta} + (1 - \lambda) E (1 + 0)^{\delta} - 1
\]
(A26)
\[
= \lambda \left( E \left( 1 + \frac{g}{R_{\delta}} \right)^{\delta} - 1 \right)
\]
\[
= \lambda.0
\]
\[
= 0.
\]
Thus,
\[
R_{\delta} [\lambda + g] = R_{\delta} [g].
\]
(A27)

Since the proof of (iv) is given, we need a formal proof for: \( \min \{ R_{\delta} [g], R_{\delta} [h] \} \leq R_{\delta} [g + h] \). We denote \( \rho = \frac{1}{R_{\delta}[g]} \) and notice that:
\[
(1 + \rho (g + h))^{\delta} = (1 + \rho g) (1 + \rho h) - \rho^2 gh
\]
(A28)
\[
= \left( (1 + \rho g) (1 + \rho h) \left( 1 - \frac{\rho^2 gh}{(1 + \rho g) (1 + \rho h)} \right) \right)^{\delta}
\]
\[
= (1 + \rho g)^{\delta} (1 + \rho h)^{\delta} \left( 1 - \frac{\rho^2 gh}{(1 + \rho g) (1 + \rho h)} \right)^{\delta}.
\]
Notice that
\[
\left( \frac{(1 - x)^{\delta} - 1}{\delta} \right) \leq -x, \forall x < 1, \delta < 0,
\]
Since
\[
\frac{1 + \rho (g + h)}{(1 + \rho g) (1 + \rho h)} > 0,
\]
it follows that
\[
1 - \frac{1 + \rho (g + h)}{(1 + \rho g) (1 + \rho h)} < 1.
\]
which is equivalent to
\[
\frac{\rho^2 gh}{(1 + \rho g) (1 + \rho h)} < 1.
\]
Therefore,
\[
\left( \frac{1 - \frac{\rho^2 gh}{(1 + \rho g) (1 + \rho h)}}{\delta} \right)^{\delta} \leq \frac{1}{\delta} - \frac{\rho^2 gh}{(1 + \rho g) (1 + \rho h)}
\]
and (A28) can be simplified to
\[
\frac{(1 + \rho (g + h))^\delta}{\delta} \leq (1 + \rho g)^\delta (1 + \rho h)^\delta \left( \frac{1}{\delta} - \frac{\rho^2 gh}{(1 + \rho g)(1 + \rho h)} \right)
\]
\[
\leq (1 + \rho g)^\delta (1 + \rho h)^\delta \frac{1}{\delta} - \left( \rho g (1 + \rho g)^{\delta-1} \right) \left( \rho h (1 + \rho h)^{\delta-1} \right).
\]
(A29)

Now, since \( g \) and \( h \) are independent, we take the expectation of (A29):
\[
E \frac{(1 + \rho (g + h))^\delta}{\delta} - \frac{1}{\delta} \leq E \frac{(1 + \rho g)^\delta E (1 + \rho h)^\delta - 1}{\delta} - \rho^2 E \left( g (1 + \rho g)^{\delta-1} \right) E \left( h (1 + \rho h)^{\delta-1} \right)
\]
(A30)

Since \( R_{\delta} [g] < R_{\delta} [h] \) and \( \rho = \frac{1}{R_{\delta} [g]} \), it follows that
\[
E \frac{(1 + \rho g)^\delta - 1}{\delta} = 0 \geq E \frac{(1 + \rho h)^\delta - 1}{\delta}
\]
(A31)

and
\[
E \left( g (1 + \rho g)^{\delta-1} \right) < 0 \text{ and } E \left( h (1 + \rho h)^{\delta-1} \right) < 0.
\]

Now, we show that
\[
E \frac{(1 + \rho g)^\delta E (1 + \rho h)^\delta - 1}{\delta} < 0.
\]
(A32)

We notice that, because \( E \frac{(1 + \rho g)^\delta - 1}{\delta} = 0 \), we have
\[
E \frac{(1 + \rho g)^\delta E (1 + \rho h)^\delta - 1}{\delta} = E \frac{(1 + \rho g)^\delta}{\delta} E (1 + \rho h)^\delta - \frac{1}{\delta}
\]
(A33)

\[
= \frac{1}{\delta} E (1 + \rho h)^\delta - \frac{1}{\delta}
\]

\[
= E (1 + \rho h)^\delta - \frac{1}{\delta}
\]

\[
= E (1 + \rho h)^\delta - \frac{1}{\delta}
\]

We, therefore, deduce from (A33):
\[
E \frac{(1 + \rho g)^\delta E (1 + \rho h)^\delta - 1}{\delta} < 0.
\]
Finally, from (A30),
\[
E \left( \frac{(1 + \rho (g+h))^{\delta} - 1}{\delta} \right) = E \left( \frac{1 + \frac{1}{R_{\delta}[g]} (g+h))^{\delta} - 1}{\delta} \right) < 0 = E \left( \frac{1 + \frac{1}{R_{\delta}[g+h]} (g+h))^{\delta} - 1}{\delta} \right).
\]

Hence
\[
R_{\delta}[g] = \min \{ R_{\delta}[g], R_{\delta}[h] \} \leq R_{\delta}[g+h].
\]

\[\square\]

**Proof of Proposition 2.** We denote and
\[
r_{0,\delta} = R_{\delta}[g] \quad \text{and} \quad u_0[x] = \frac{(1 + x/r_{0,\delta})^{\delta} - 1}{\delta}.
\]

If \(g \sim_{SD_1} h\) or \(g \sim_{SD_2} h\), then \(E u_0[g] > E u_0[h]\) since \(u_0[x]\) is strictly monotonic and strictly concave. However \(E u_0[g] = 0\) from Theorem 1. Therefore \(E u_0[h] < 0\), that is,
\[
E \left( \frac{(1 + h/r_{0,\delta})^{\delta} - 1}{\delta} \right) < \frac{E (1 + h/R_{\delta}[h])^{\delta} - 1}{\delta} \quad \text{which implies}
\]
\[
R_{\delta}[g] = r_{0,\delta} < R_{\delta}[h].
\]

\[\square\]

**Proof of Proposition 3.** We first show the following lemma:

**Lemma 1** Let \(\{g_n\}_{n=1,2,\ldots} \subset G\) be a sequence of gambles satisfying
\[
\sup_{n \geq 1} M[g_n] < \infty.
\]

If \(g_n \stackrel{p}{\rightarrow} g \in G\) and \(L[g_n] \rightarrow L[g]\) as \(n \rightarrow \infty\), then \(R_{\delta}[g_n] \rightarrow \max \{ R_{\delta}[g], L_0 \}\) as \(n \rightarrow \infty\).

**Proof.** We denote
\[
R_{0,\delta} = \max \{ R_{\delta}[g], L_0 \}
\]

Using Theorem 2.1 (iv) in Billingsley (1968), \(g_n \stackrel{p}{\rightarrow} g\) implies
\[
\liminf_{n} L[g_n] \geq L[g].
\]
Because
\[ \liminf_n P[g_n < -L[g] + \varepsilon] \geq P[g < -L[g] + \varepsilon] > 0 \quad \forall \varepsilon > 0, \]

and
\[ L_0 \geq L[g]. \quad \text{(A34)} \]

It follows that
\[ r \geq L_0 \quad \text{(A35)} \]

where
\[ R_\delta[g_n] \to r \text{ as } n \to \infty \]

because
\[ R_\delta[g_n] > L[g_n] \to L_0. \]

Now, we will show that \( r \geq R_\delta[g] \).

Let \( 0 < \varepsilon < 1 \) and \( q = (1 + \varepsilon)^2 r \). Then,

\[ R_\delta[g_n] < q \quad \forall n \quad \text{(A36)} \]

and
\[ L[g_n] < (1 + \varepsilon)L_0. \quad \text{(A37)} \]

To obtain (A37), notice that
\[ |L[g_n] - L_0| < \zeta \quad \forall \zeta > 0. \]

Setting
\[ \zeta = \varepsilon L_0, \]

produces (A37). Therefore,
\[ (1 + \varepsilon)L_0 \leq (1 + \varepsilon)r = \frac{1}{1+\varepsilon}q \]

and
\[ L[g_n] < (1 + \varepsilon)L_0 \leq (1 + \varepsilon)r = \frac{1}{1+\varepsilon}q. \quad \text{(A38)} \]
Using Lemma 1 and (A36), it follows that

\[ E \left( \frac{1 + g_n/q}{\delta} \right)^{\delta} - 1 < 0. \]

In addition, \( \frac{E(1+g_n/q)^{\delta}-1}{\delta} \) is uniformly bounded from above by \( \frac{(1+M|g_n|/q)^{\delta}-1}{\delta} \) and from below by \( \frac{(\pi)^{\delta}-1}{\delta} \) because from (A38), it follows that \( g_n/q \geq L[g_n] / q > \frac{1}{1+\epsilon} \). Since, \( g_n \xrightarrow{P} g \in G \),

\[ \frac{E(1+g/q)^{\delta}-1}{\delta} = \lim_{n} \frac{E(1+g_n/q)^{\delta}-1}{\delta} \geq 0 = \frac{E(1+g/R_0[g])^{\delta}-1}{\delta}. \]

Therefore, by Lemma 1,

\[ q = (1+\epsilon)^2 r \geq R_0[g]. \quad (A39) \]

Since \( \epsilon \) is arbitrary, it follows that

\[ r \geq R_0[g]. \quad (A40) \]

Equations (A35) and (A39) imply that

\[ r \geq \max \{ R_0[g], L_0 \} = R_{0,\delta}. \]

Now, we will show that, it is impossible to get \( r > R_{0,\delta} \). To proceed, assume that \( r > R_{0,\delta} \) and consider \( 0 < \epsilon < 1 \) small enough so that

\[ q = (1+\epsilon)^2 R_{0,\delta} < r. \]

For large values \( n \), we have

\[ q < R_0[g_n], \quad (A41) \]

and

\[ L(g_n) < (1+\epsilon) L_0 \leq (1+\epsilon) R_{0,\delta} = \frac{1}{1+\epsilon} q. \quad (A42) \]

Lemma 1 and (A41) allows to write

\[ E(1+g_n/q)^{\delta}-1 \delta < 0. \]

Since \( \frac{(1+g_n/q)^{\delta}-1}{\delta} \) is bounded from below by (A42), it follows from \( g_n \xrightarrow{P} g \) that

\[ \frac{E(1+g/q)^{\delta}-1}{\delta} = \lim_{n} \frac{E(1+g_n/q)^{\delta}-1}{\delta} \leq 0. \quad (A43) \]
Equation (A43) implies that \( \frac{1}{q} > \frac{1}{R_0} \), which contradicts

\[
q = (1 + \varepsilon)^2 R_{0,\delta} \geq (1 + \varepsilon)^2 R_\delta[g] > R_\delta[g].
\]

Therefore \( r = R_{0,\delta} \), that is

\[
R_\delta[g_n] \to R_{0,\delta} \text{ as } n \to \infty.
\]

If \( L_0 = L[g] \), \( R_{0,\delta} = \max \{ R_\delta[g], L_0 \} = R_\delta[g] \). Hence, from (A44),

\[
R_\delta[g_n] \to R_\delta[g] \text{ as } n \to \infty.
\]
B. SIMPLE RETURNS

In Theorem 1, we show that there exists a critical wealth level \( R_{\delta,t} [g_{t+\tau}] \) such as

\[
E_t \left( 1 + \frac{g_{t+\tau}}{R_{\delta,t} [g_{t+\tau}]} \right)^\delta - 1 = 0. 
\]  

**UNDER THE RISK NEUTRAL MEASURE**, (B1) can be expressed as:

\[
E_t^* \left( 1 + \frac{g_{t+\tau}}{R_{\delta,t} [g_{t+\tau}]} \right)^\delta - 1 = 0. 
\]  

We denote

\[
g_{t+\tau} = \frac{S_i(t, \tau) - S_i(t)}{S_i(t)} 
\]  

the return on the risky asset \( i \) with an investment horizon \( \tau \). Notice that, under the risk neutral measure

\[
E_t^* [g_{t+\tau}] = r_f (t, \tau), 
\]  

where \( r_f (t, \tau) \) represents the risk-free rate for the time period \([t, t+\tau]\). Since \( \left( \frac{E_t^* \left( 1 + \frac{g_{t+\tau}}{R_{\delta,t} [g_{t+\tau}]} \right)^\delta - 1}{\delta} \right) \) is finite,

we can use the Bakshi and Madan (2000) spanning formula:

\[
G[S] = G[S] + (S - S) G_s[S] + \int_S^\infty G_{SS}[K] (S - K)^{+} dK + \int_0^S G_{SS}[K] (K - S)^+ dK 
\]  

Now, we consider the function

\[
G[S_i(t, \tau)] = \left( 1 + \frac{g_{t+\tau}}{R_{\delta,t} [g_{t+\tau}]} \right)^\delta 
\]  

with \( S = S_i(t) \). Therefore, applying (B5) to (B6) gives:

\[
\left( 1 + \frac{g_{t+\tau}}{R_{\delta,t} [g_{t+\tau}]} \right)^\delta - 1 = (S_i(t, \tau) - S_i(t)) \frac{1}{S_i(t) R_{\delta,t} [g_{t+\tau}]} 
\]

\[
+ \int_{S_i(t)}^\infty H_{ss}[K] (S_i(t, \tau) - K)^{+} dK 
\]

\[
+ \int_0^{S_i(t)} H_{ss}[K] (K - S_i(t, \tau))^{+} dK, 
\]
where $H_{ss}[K]$ is given by:

$$H_{ss}[K] = \frac{(\delta - 1)}{(S_i(t) R_{\delta,t}[gt+\tau])^2} \left( 1 + \frac{K_{S(t)} - 1}{R_{\delta,t}[gt+\tau]} \right)^{\delta-2}. \quad (B8)$$

We apply the expectation operator under the risk neutral measure to (B7):

$$r_f(t, \tau) \frac{1}{R_{\delta,t}[gt+\tau]} = -\int_{S_i(t)}^{\infty} H_{ss}[K] E^*_t (S_i(t, \tau) - K)^+ dK - \int_{0}^{S_i(t)} H_{ss}[K] E^*_t (K - S_i(t, \tau))^+ dK. \quad (B9)$$

We recall that the price of the call and put options with strike $K$ and maturity $\tau$ are given by (B10) and (B11) respectively

$$\frac{1}{(1+r_f(t, \tau))} E^*_t (S_i(t, \tau) - K)^+ = C(S_i(t), K, \tau), \quad (B10)$$

$$\frac{1}{(1+r_f(t, \tau))} E^*_t (K - S_i(t, \tau))^+ = P(S_i(t), K, \tau), \quad (B11)$$

where $(1+r_f(t, \tau))$ represents the risk-free return for the time period $[t, t+\tau]$. Hence, (B9) reduces to

$$r_f(t, \tau) \frac{1}{1+r_f(t, \tau) R_{\delta,t}[gt+\tau]} = \int_{S_i(t)}^{\infty} f_R[K] C(S_i(t), K, \tau) dK + \int_{0}^{S_i(t)} f_R[K] P(S_i(t), K, \tau) dK \quad (B12)$$

with

$$f_R[K] = \frac{(1-\delta)}{(S_i(t) R_{\delta,t}[gt+\tau])^2} \left( 1 + \frac{K_{S(t)} - 1}{R_{\delta,t}[gt+\tau]} \right)^{\delta-2}. \quad (B13)$$

Equation (B12) can be numerically solved to deduce critical wealth level $R_{\delta,t}[gt+\tau]$ (solve for the fixed point $f(x) = x$).
C. LOG RETURNS

Consider the log return
\[ g_{t+\tau} = \log \frac{S_i(t+\tau)}{S_i(t)}, \]

We obtain
\[
\left( 1 + \frac{g_{t+\tau}}{R_{\delta,t}[g_{t+\tau}]} \right)^{\delta} - 1 = \left( S_i(t+\tau) - S_i(t) \right) \left( \frac{1}{R_{\delta,t}[g_{t+\tau}]} \right) \]
\[ + \int_{S_i(t)}^{\infty} H_{ss}[K] (S_i(t+\tau) - K)^+ dK \]
\[ + \int_{0}^{S_i(t)} H_{ss}[K] (K - S_i(t+\tau))^+ dK \]
\]

where \( H_{ss}[K] \) is given by
\[
H_{ss}[K] = \left( 1 + \frac{\log K}{R_{\delta,t}[S_i(t+\tau)]} \right)^{\delta-2} \left( \frac{1}{K^2} \left( \frac{\delta}{\delta^2} - 1 \right) - \frac{\log K}{R_{\delta,t}[S_i(t+\tau)]} \right).
\]

Applying the expectation operator under the risk neutral measure to (C1) gives:
\[
\frac{r_f(t,\tau)}{(1+r_f(t,\tau)) R_{\delta,t}[g_{t+\tau}]} = \int_{S_i(t)}^{\infty} f_R[K] C(S_i(t),K,\tau) dK + \int_{0}^{S_i(t)} f_R[K] P(S_i(t),K,\tau) dK \]
\[ \text{(C2)} \]

with
\[
f_R[K] = \left( 1 + \frac{\log K}{R_{\delta,t}[S_i(t+\tau)]} \right)^{\delta-2} \left( \frac{1}{K^2} \left( \frac{\delta}{\delta^2} - 1 \right) + \frac{\log K}{R_{\delta,t}[S_i(t+\tau)]} \right).
\]
\[ \text{(C3)} \]

Equation (C2) can be solved numerically to recover \( R_{t}[g_{t+\tau}] \) (solve for the fixed point \( f[x] = x \)).
II. Riskiness Measure and Utility Function

Consider an investor with constant relative risk aversion (CRRA) utility function

\[ u(W) = \frac{W^\delta}{\delta} \]  

The representative investor with utility (1) will reject the gamble if, \( E_t u(W_t + g_{t+1}) < u(W_t) \),

\[ \frac{E_t \left(1 + \frac{g_{t+1}}{W_t}\right)^\delta}{\delta} - 1 < 0. \]  

Since the function \( \phi(\delta)[x] \) in equation (3)

\[ \phi(\delta)[x] = \begin{cases} \frac{E_t(1+xg_{t+1})^\delta}{\delta} - 1 & \text{if } \delta < 0 \\ E_t \left( \log \left(1 + xg_{t+1}\right) \right) & \text{if } \delta = 0 \end{cases} \]  

is concave and that our riskiness measure is the unique solution to \( \phi(\delta)[x] = 0 \), inequality (2) implies that

\[ \frac{1}{R_{1.0}[g_{t+1}]} < \frac{1}{W_t}. \]  

Therefore, the representative agent will reject the gamble \( g_{t+1} \) if her current wealth is below \( R_{\delta,t}[g_{t+1}] \).

Merton’s (1973) intertemporal capital asset pricing model (ICAPM) indicates that the conditional expected excess return on a risky market portfolio is a linear function of its conditional variance assuming that hedging demands are not too large:

\[ E_t \left( R_{m,t+1} \right) = \theta \cdot E_t \left( \sigma_{m,t+1}^2 \right), \]  

where \( E_t \left( R_{m,t+1} \right) \) and \( E_t \left( \sigma_{m,t+1}^2 \right) \) are, respectively, the conditional mean and variance of excess returns on the market portfolio, and \( \theta = 1 - \delta > 0 \) is the relative risk aversion of market investors. Equation (4) establishes the dynamic relation that investors require a larger risk premium at times when the market is riskier.

The following GARCH-in-Mean process is used to estimate the intertemporal relation between ex-
Expected return and risk on the stock market portfolio:

\[ R_{m,t+1} = \omega + \theta \sigma^2_{m,t+1|t} + \epsilon_{m,t+1}, \quad (5) \]

\[ E \left( \epsilon^2_{m,t+1|\Omega_t} \right) = \sigma^2_{m,t+1|t} = \gamma_0 + \gamma_1 \epsilon^2_{m,t} + \gamma_2 \sigma^2_{m,t}, \quad (6) \]

where \( R_{m,t+1} \) is the daily excess return on the market portfolio for time \( t+1 \), \( \Omega_t \) denotes information set up to time \( t \), \( E \left( R_{m,t+1|\Omega_t} \right) = \omega + \theta \sigma^2_{m,t+1|t} \) is the conditional expected excess return on the market portfolio for time \( t+1 \) based on \( \Omega_t \), \( \sigma^2_{m,t+1|t} \) is the conditional variance of excess market returns for time \( t+1 \) based on \( \Omega_t \). The conditional variance, \( \sigma^2_{m,t+1|t} \), is defined as a function of the last period’s unexpected news (or information shocks), \( \epsilon_{m,t} \), and the last period’s variance, \( \sigma^2_{m,t} \). Our focus is to examine the magnitude and statistical significance of the relative risk aversion parameter \( \theta \) in equation (5).

In our empirical analyses, we use four different stock market indices to proxy for the market portfolio: (i) the value-weighted NYSE/AMEX/NASDAQ index, also known as the value-weighted index of the Center for Research in Security Prices (CRSP), can be viewed as the broadest possible stock market index used in earlier studies, (ii) New York Stock Exchange (NYSE) index, (iii) Standard and Poor’s 500 (S&P 500) index, and (iv) Dow Jones Industrial Average (DJIA) can be viewed as the smallest possible stock market index used in earlier studies. We use the longest common sample period from July 1, 1963 to December 31, 2009, for a total of 11,707 daily observations.

As presented in Table I of the online appendix, the risk aversion parameter (\( \theta \)) is estimated to be positive and statistically significant for all market indices. Specifically, \( \theta \) is estimated to be in the range of 3.05 to 3.17 with the Bollerslev and Wooldridge (1992) t-statistics ranging from 2.43 to 2.63, indicating a positive and strong intertemporal relation between expected return and risk on the market. Based on the relative risk aversion interpretation, the magnitudes of these estimates are economically sensible and consistent with the findings of earlier studies (e.g., Bali (2008) and Guo and Whitelaw (2006)).

To gain some insight about the economic significance of risk aversion parameter, we compute \( \theta \sigma^2 \) using the aforementioned estimates of \( \theta \) and the unconditional variance of the market portfolio \( \sigma^2 \). For the sample period of 1963-2009, the standard deviation of excess market returns is in the range of 0.96% to 1% per day. Since \( \theta \) is in the range of 3.05 to 3.17, the expected market risk premium is approximately 7% per annum assuming 252 trading days in a year. To check whether this implied expected market

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1 The GARCH-in-Mean models are originally introduced by Engle, Lilien, and Robins (1987) and then used by a number of studies including Engle, Ng, and Rothschild (1990), Nelson (1991), and many recent papers on risk-return tradeoff.
risk premium is close to its empirical counterpart, we compute the average excess return on the market portfolio, which is close to 6% per annum for the sample period of 1963-2009. These results provide evidence that the estimated value of relative risk aversion coefficient is not only positive and significant, but it is economically meaningful as well.

III. The Generalized Physical Measure of Riskiness

In Section VII, we present results from the generalized measure of physical riskiness. To be consistent with the options’ implied measure of riskiness, in Section VII we first compute the mean, standard deviation, and skewness of daily returns over the past 1, 3, 6, and 12 months and then numerically back out the physical measure of generalized riskiness from equation (33) with $\delta = -2$. Figure 3 in the online appendix presents the generalized measure of riskiness obtained from the physical distribution proxied by the mean, standard deviation, and skewness of daily returns on the S&P 500 index. Since the average daily returns over the past 1, 3, 6, and 12 months are negative for some periods in our sample (1996-2008), the physical riskiness measure can potentially be negative for some periods.

As shown in Figure 3, the generalized measure of physical riskiness obtained from the S&P 500 index turns negative during a significant part of the sample period. To generate an alternative measure of physical riskiness, instead of using the sample average return (i.e., the first moment of the empirical return distribution), we use a constant, positive expected rate of return on the S&P 500 index, 6% per annum (i.e., $\mu = 6\%$ in equation (33)). Figure 4 of the online appendix plots this alternative measure of riskiness for the sample period January 1996 - December 2008. Since we impose a positive expected market risk premium, physical riskiness becomes a function of standard deviation and skewness, and as shown in Figure 4, physical riskiness is now positive throughout the sample period.

To generate an alternative measure of physical riskiness for each stock in our sample, instead of using the average daily returns over the past 1, 3, 6, and 12 months, we use an expected return based on the capital asset pricing model (CAPM):

$$E(R_i) = r_f + \beta_i [E(R_m) - r_f]$$

(7)

where $r_f$ is the risk-free interest rate (proxied by the 3-month T-bill rate), $E(R_m)$ is the expected return on
the market portfolio, which is assumed to be constant at 6% to be consistent with Figure 4, and \( \beta_i \) is the market beta of stock \( i \), computed using daily returns on stock \( i \) and the market portfolio over the past 1, 3, 6, and 12 months. Once we estimate \( E(R_i) \) for each stock \( i \), we use it for \( \mu \) in equation (33).

We test the predictive power of this alternative measure of physical riskiness using the Fama and MacBeth cross-sectional regressions:

\[
\frac{g_{it+1} - r_{ft+1}}{\sigma_{it+1}} = a_{0t} + a_{1t} R_{S,t}^P [g_{it+1}] + \epsilon_{it+1}
\]

where \( g_{it+1} \) is estimated using the average and holding period returns for the past 1, 3, 6, and 12 months. Similarly, \( \sigma_{it+1} \) is estimated using daily returns over the past 1, 3, 6, and 12 months and \( r_{ft+1} \) is proxied by the 3-month Treasury bill rate. \( R_{S,t}^P [g_{it+1}] \) is the generalized physical measure of riskiness of stock \( i \) in month \( t \) estimated with \( \mu_i = E(R_i) \) in equation (7). \( a_{0t} \) and \( a_{1t} \) are the monthly intercepts and slope coefficients from the Fama-MacBeth regressions, respectively.

Table II of the online appendix shows that the average slopes on the alternative physical measure of riskiness are negative, but they are not statistically significant for all horizons. This result holds for both the average and the holding period return definition of the Sharpe ratios, providing no evidence for a robust, significant link between the physical measure of riskiness and the cross-section of risk-adjusted returns. However, the alternative measure of physical riskiness provides stronger results compared to those reported in Table 4.

We also investigate whether the physical measure of riskiness is able to rank stocks based on their expected returns per unit of risk. Quintile portfolios are formed every month from January 1996 to September 2008 by sorting individual stocks based on this alternative measure of physical riskiness. The results reported in Table III of the online appendix indicate a significantly negative link between physical riskiness and future risk-adjusted returns for 1-month and 6-month investment horizons, whereas the cross-sectional relation is weak for 3-month and 12-month horizons.

Finally, we examine the predictive power of the riskiness premium defined based on the alternative measure of physical riskiness. We form quintile portfolios every month from January 1996 to September 2008 by sorting individual stocks based on the spread \( R_{S,t}^Q [g_{it+1}] - R_{S,t}^P [g_{it+1}] \). Table IV of the online appendix shows that when moving from Low \( R_{S,t}^Q - R_{S,t}^P \) to High \( R_{S,t}^Q - R_{S,t}^P \) portfolios, there is a significant decline in risk-adjusted returns of quintile portfolios, indicating a negative relation between the riskiness
premium and the expected excess returns per unit of risk. Another notable point in Table IV is that the differences in risk-adjusted returns between quintiles 5 and 1 are negative without any exception. The last row of Table 6 shows that these risk-adjusted return differences are also highly significant with the Newey-West t-statistics ranging from -2.39 to -3.57.\textsuperscript{3} Overall, these results are similar to our earlier findings in Table 6.

\textsuperscript{3}The only exception is the 12-month average return definition of the Sharpe ratio for which the Newey-West t-statistic is -1.52.
References


Table I. Estimating Risk Aversion with the GARCH-in-Mean Model
This table presents the maximum likelihood estimates of the GARCH-in-Mean model for the alternative stock market indices, 

\[ R_{m,t+1} = \omega + \theta \sigma^2_{m,t+1|t} + \epsilon_{m,t+1}, \]

\[ E(\epsilon^2_{m,t+1|\Omega_t}) = \gamma_0 + \gamma_1 \epsilon^2_{m,t} + \gamma_2 \sigma^2_{m,t}, \]

where \( R_{m,t+1} \) is the daily excess return on the market portfolio for time \( t + 1 \), \( \Omega_t \) denotes information set up to time \( t \), \( E(R_{m,t+1|\Omega_t}) = \omega + \theta \sigma^2_{m,t+1|t} \) is the conditional expected excess return on the market portfolio for time \( t + 1 \) based on \( \Omega_t \), \( \sigma^2_{m,t+1|t} \) is the conditional variance of excess market returns for time \( t + 1 \) based on \( \Omega_t \). The conditional variance, \( \sigma^2_{m,t+1|t} \) is defined as a function of the last period’s unexpected news (or information shocks), \( \epsilon_{m,t} \), and the last period’s variance \( \sigma^2_{m,t} \). The parameters are estimated using daily returns for the sample period of July 1, 1963 to December 31, 2009, for a total of 11,707 observations. The Bollerslev and Wooldrige (1992) robust t-statistics are given in parentheses.

<table>
<thead>
<tr>
<th>Market Portfolio</th>
<th>( \omega )</th>
<th>( \theta )</th>
<th>( \gamma_0 )</th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NYSE/AMEX/NASDAQ</td>
<td>0.00028</td>
<td>3.0882</td>
<td>6.70 \times 10^{-7}</td>
<td>0.0856</td>
<td>0.9096</td>
</tr>
<tr>
<td></td>
<td>(3.40)</td>
<td>(2.53)</td>
<td>(4.39)</td>
<td>(8.54)</td>
<td>(112.05)</td>
</tr>
<tr>
<td>NYSE</td>
<td>0.00027</td>
<td>3.1482</td>
<td>6.43 \times 10^{-7}</td>
<td>0.0833</td>
<td>0.9121</td>
</tr>
<tr>
<td></td>
<td>(3.17)</td>
<td>(2.52)</td>
<td>(3.39)</td>
<td>(8.16)</td>
<td>(118.05)</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>0.00010</td>
<td>3.0520</td>
<td>5.08 \times 10^{-7}</td>
<td>0.0744</td>
<td>0.9232</td>
</tr>
<tr>
<td></td>
<td>(1.15)</td>
<td>(2.63)</td>
<td>(3.62)</td>
<td>(7.25)</td>
<td>(114.82)</td>
</tr>
<tr>
<td>Dow Jones</td>
<td>0.00006</td>
<td>3.1692</td>
<td>7.55 \times 10^{-7}</td>
<td>0.0700</td>
<td>0.9243</td>
</tr>
<tr>
<td></td>
<td>(0.56)</td>
<td>(2.43)</td>
<td>(3.44)</td>
<td>(5.70)</td>
<td>(98.86)</td>
</tr>
</tbody>
</table>
Table II. Cross-Sectional Regressions of Risk-Adjusted Returns on the Generalized Physical Measure of Riskiness

This table reports the average intercept and slope coefficients from the Fama and MacBeth (1973) cross-sectional regressions of one-month ahead risk-adjusted returns of individual stocks on the stocks’ generalized physical measure of riskiness,

\[
\frac{g_{i,t+1} - r_{f,t+1}}{\sigma_{i,t+1}} = a_{0,t} + a_{1,t} R_{g,t}^P [g_{i,t+1}] + \epsilon_{i,t+1}
\]

where \( g_{i,t+1} \) is the average (or holding period) return on stock \( i \) in month \( t + 1 \), is the risk-free interest rate in month \( t + 1 \), \( g_{i,t+1} - r_{f,t+1} \) is the expected excess return on stock \( i \) in month \( t + 1 \), \( \sigma_{i,t+1} \) is the standard deviation of stock \( i \) in month \( t + 1 \), and \( \frac{g_{i,t+1} - r_{f,t+1}}{\sigma_{i,t+1}} \) is the risk-adjusted return of stock \( i \) in month \( t + 1 \). \( R_{g,t}^P [g_{i,t+1}] \) is the generalized physical \( P \) measure of riskiness of stock \( i \) in month \( t \) obtained from daily returns over the past 1, 3, 6, and 12 months. \( a_{0,t} \) and \( a_{1,t} \) are the monthly intercepts and slope coefficients from the Fama-MacBeth regressions. In the first stage, the generalized measure of riskiness is estimated using equation (33). The risk-adjusted returns of individual stocks are estimated using average and holding period returns along with the standard deviations from the past 1, 3, 6, and 12 months of daily data. In the second stage, the cross-section of one-month ahead risk-adjusted returns are regressed on the physical measures of riskiness each month from January 1996 to September 2008. Newey and West (1987) t-statistics are reported in parentheses.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Average Returns</th>
<th>Holding Period Return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \bar{a}_0 )</td>
<td>( \bar{a}_1 )</td>
</tr>
<tr>
<td>1-month</td>
<td>0.0968</td>
<td>-6.9331</td>
</tr>
<tr>
<td></td>
<td>(2.78)</td>
<td>(-3.84)</td>
</tr>
<tr>
<td>3-month</td>
<td>0.0719</td>
<td>-0.1179</td>
</tr>
<tr>
<td></td>
<td>(2.14)</td>
<td>(-0.97)</td>
</tr>
<tr>
<td>6-month</td>
<td>0.0740</td>
<td>-0.1896</td>
</tr>
<tr>
<td></td>
<td>(2.20)</td>
<td>(-1.39)</td>
</tr>
<tr>
<td>12-month</td>
<td>0.0815</td>
<td>-0.2772</td>
</tr>
<tr>
<td></td>
<td>(2.42)</td>
<td>(-2.10)</td>
</tr>
</tbody>
</table>
Table III. Risk-Adjusted Returns of Quintile Portfolios Formed Based on the Generalized Physical Measure of Riskiness

Quintile portfolios are formed every month from January 1996 to September 2008 by sorting individual stocks based on their generalized physical measure of riskiness defined in equation (33). Quintile 1 (Low \( R^P_{g_i,t+1} \)) is the portfolio of stocks with the lowest riskiness and Quintile 5 (High \( R^P_{g_i,t+1} \)) is the portfolio of stocks with the highest riskiness. The table reports the next month expected excess returns per unit of risk, where the expected return is measured by the average and holding period returns for the past 1, 3, 6, and 12 months and risk is measured by the standard deviation of daily returns over the past 1, 3, 6, and 12 months. The last row shows the differences in expected returns per unit of risk between High \( R^P_{g_i,t+1} \) and Low \( R^P_{g_i,t+1} \) portfolios. Newey-West adjusted t-statistics are given in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Average Return</th>
<th>Holding Period Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-month</td>
<td>3-month</td>
</tr>
<tr>
<td>Low ( R^P_{g_i,t+1} )</td>
<td>( 0.0413 )</td>
<td>( 0.0175 )</td>
</tr>
<tr>
<td>2</td>
<td>( 0.0798 )</td>
<td>( 0.0775 )</td>
</tr>
<tr>
<td>3</td>
<td>( 0.0399 )</td>
<td>( 0.0578 )</td>
</tr>
<tr>
<td>4</td>
<td>( 0.0320 )</td>
<td>( 0.0222 )</td>
</tr>
<tr>
<td>High ( R^P_{g_i,t+1} )</td>
<td>( -0.0133 )</td>
<td>( -0.0002 )</td>
</tr>
<tr>
<td>High-Low</td>
<td>( -0.0546 )</td>
<td>( -0.0177 )</td>
</tr>
</tbody>
</table>

\((-3.20)\) \((-1.47)\) \((-2.11)\) \((-1.97)\) \((-3.20)\) \((-0.85)\) \((-2.01)\) \((-1.84)\)
Quintile portfolios are formed every month from January 1996 to September 2008 by sorting individual stocks based on the riskiness premium, defined as the difference between the generalized option’s implied and the generalized physical measures of riskiness, $R^{O}_{\delta,t} [g_{i,t+1}] - R^{P}_{\delta,t} [g_{i,t+1}]$. Quintile 1 (Low $R^{O}_{\delta,t} - R^{P}_{\delta,t}$) is the portfolio of stocks with the lowest riskiness premium and Quintile 5 (High $R^{O}_{\delta,t} - R^{P}_{\delta,t}$) is the portfolio of stocks with the highest riskiness premium. The table reports the next month expected excess returns per unit of risk, where the expected return is measured by the average and holding period returns for the past 1, 3, 6, and 12 months and risk is measured by the standard deviation of daily returns over the past 1, 3, 6, and 12 months. The last row shows the differences in expected returns per unit of risk between High $R^{O}_{\delta,t} - R^{P}_{\delta,t}$ and Low $R^{O}_{\delta,t} - R^{P}_{\delta,t}$ portfolios. Newey-West adjusted t-statistics are given in parentheses.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>1-month</th>
<th>3-month</th>
<th>6-month</th>
<th>12-month</th>
<th>1-month</th>
<th>3-month</th>
<th>6-month</th>
<th>12-month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low $R^{O}<em>{\delta,t} - R^{P}</em>{\delta,t}$</td>
<td>0.0338</td>
<td>0.0601</td>
<td>0.0291</td>
<td>0.0354</td>
<td>0.0338</td>
<td>0.0636</td>
<td>0.0681</td>
<td>0.0895</td>
</tr>
<tr>
<td>2</td>
<td>0.0649</td>
<td>0.0930</td>
<td>0.0695</td>
<td>0.0674</td>
<td>0.0649</td>
<td>0.0947</td>
<td>0.1127</td>
<td>0.1384</td>
</tr>
<tr>
<td>3</td>
<td>0.0653</td>
<td>0.0689</td>
<td>0.0721</td>
<td>0.0806</td>
<td>0.0653</td>
<td>0.1060</td>
<td>0.1217</td>
<td>0.1745</td>
</tr>
<tr>
<td>4</td>
<td>0.0599</td>
<td>0.0183</td>
<td>0.0426</td>
<td>0.0588</td>
<td>0.0599</td>
<td>0.0609</td>
<td>0.0705</td>
<td>0.1187</td>
</tr>
<tr>
<td>High $R^{O}<em>{\delta,t} - R^{P}</em>{\delta,t}$</td>
<td>-0.0326</td>
<td>-0.0443</td>
<td>-0.0210</td>
<td>0.0079</td>
<td>-0.0326</td>
<td>-0.0504</td>
<td>-0.0532</td>
<td></td>
</tr>
<tr>
<td>High-Low</td>
<td>-0.0664</td>
<td>-0.1045</td>
<td>-0.0501</td>
<td>-0.0274</td>
<td>-0.0664</td>
<td>-0.1140</td>
<td>-0.1337</td>
<td>-0.1427</td>
</tr>
</tbody>
</table>

(-2.50) (-3.57) (-2.39) (-1.52) (-2.50) (-3.16) (-2.99) (-2.90)
This figure presents the time-varying investment choice of a market investor with a relative risk aversion of three over the sample period of 1996-2008. For investment horizons of 1, 3, 6, and 12 months, the figure displays a fraction of the S&P 500 index that the market investor would buy to avoid bankruptcy or extremely large losses.
Fig. 2: Options’ Implied Measures of Riskiness

This figure presents the generalized options’ implied measure of riskiness with $\delta = -2$ and the options’ implied riskiness measure of Foster and Hart (2009) with $\delta = 0$. The riskiness measures are obtained from the S&P 500 index options with 1, 3, 6, and 12 months to maturity for the sample period of January 1996 to September 2008.
Fig. 3: The Generalized Physical Measure of Riskiness
This figure presents the generalized physical measure of riskiness with $\delta = -2$ for the sample period of January 1996 to September 2008. The physical measures are obtained from daily returns on the S&P 500 index over the past 1, 3, 6, and 12 months.
Fig. 4: Alternative Measures of Physical Riskiness
This figure presents alternative measures of physical riskiness with $\delta = -2$ for the sample period of January 1996 to September 2008. The physical measures are obtained from daily returns on the S&P 500 index over the past 1, 3, 6, and 12 months and using $\mu_i = E(R_i)$ as defined in equation (7) of the online appendix.