August 15, 2000

Effects of Price Limits on Information Revelation:

Theory and Evidence

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Abstract: Many stock exchanges have daily price limits for individual stocks. The effects of these price limits are little understood, especially for price revelation and thus resource allocation. This paper models and tests how price limits may induce an informed investor to shift part or all of her profit-motivated trades until the next day, thus retarding the spread of information. The model implies these delays are particularly likely if the current price is near, but the equilibrium price is substantially beyond, today's limit. In a series of tests on daily open, close, high, low and limit prices from the Taiwan Stock Exchange, results are consistent with the model; empirical results support the view both that informed investors' trades play a major role in price revelation and that price limits importantly delay price revelation.

We thank seminar participants at the SEC's Office of Economic Analysis, the University of Wisconsin-Milwaukee, the 1999 FMA meetings and the McDonough School of Business for comments on earlier versions of this paper. Particular thanks are due to Bruce Bender, Jim Bodurtha, Lynn Doran, Lee Pinkowitz, and Eric Sirri. Parts of this study were written while Kim was a Visiting Scholar and Sweeney was a consultant with the U.S. Securities and Exchange Commission. The Securities and Exchange Commission, as a matter of policy, disclaims responsibility for any private publication or statement by any of its employees; the views here are the authors' and do not necessarily reflect the views of the Commission or its staff. Sweeney's work was partially supported by summer grants from Georgetown University and The McDonough School of Business; The Capital Markets Research Center provided summer research assistance.

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1. Introduction

Many financial asset markets have daily price limits on individual assets. U.S. futures markets are perhaps the best-known example, but price limits are common in non-U.S. equities markets, including Austria, Belgium, France, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Spain, Switzerland, Taiwan, and Thailand. Though daily price limits on individual stocks affect a significant portion of the world's capital, little is known about how these price limits affect markets and market participants' behavior, as Harris (1998) contends.

Stock markets offer little explicit justification for price limits. For example, the Taiwan Stock Exchange says it imposes price limits "… in order to maintain a stable stock market …" Other possible benefits might be: to reduce price volatility; to give participants a "time out" to digest information; to prevent bubbles. On the cost side, liquidity traders may not be able to buy or sell stocks when they had planned; further, managers following hedging strategies may be unable to rebalance portfolios when their strategies require it.

This paper builds a simple model that shows how price limits affect an informed investor's behavior and retard the flow of information that her trades reveal to the market, and thus adversely affect resource allocation. Tests on Taiwan Stock Exchange (TSE) data support the model's implications.

Only two papers model price limits on individual assets. Brennan (1986) develops a theoretical model of why price limits exist in U.S. futures markets; he predicts they will eventually disappear in most markets. Kodres and O'Brien (1994) model conditions where price limits may be useful, though limits are not very binding in their model.

Empirical literature on price limits is scant, as Harris (1998) notes. Price limit research on U.S. futures markets often uses only one or a few contracts (e.g., Kuserk, et al. (1989); Ma, Rao, and Sears (1989a,b)). To examine price limit effects on stocks, researchers turn to non-U.S. markets: Chen (1993) studies the Taiwan Stock Exchange, Kim and Rhee (1997) the Tokyo market, and Phylaktis, Kavussanos, and Manalis (1999) the Athens exchange. Empirical price-limit research on U.S. futures and non-U.S. equities investigates two main questions: do price limits reduce volatility, and do they mitigate investor overreaction? Ma, Rao, and Sears (1989a,b), answer "yes" to both; Lehmann (1989) and Miller (1989) point out problems with these studies, however, that subsequent papers overcome. In later work, Chen (1993), Chen (1998), Kim and Rhee (1997), and Phylaktis, et al. (1999), answer "no."

Our paper differs importantly from prior empirical work by focusing on the informed investor. Ours is the first paper to model and then test how price limits affect the informed investor's trading strategy and thus the timing and quantity of information her trades reveal. Previous empirical papers are a-theoretical, say nothing systematic about the informed investor, and do not test propositions about price-limit effects on the informed investor and her information revelation. Further, previous empirical work assesses price-limit effects primarily by studying stocks that hit their limits. In contrast, our model predicts that those stocks near but not at their limits contain crucial information on price-limit effects, and develops predictions about how these stocks' prices will behave. Our tests validate these predictions about price behavior.

The literatures on (1) regularly scheduled market closes, (2) circuit breakers, and (3) trading halts can provide some insights, by analogy, into price limits on individual stocks, but the insights are limited. Market closes, circuit breakers, trading halts and price limits all require separate, explicit modeling and testing.

The effects of regularly scheduled market closes on trading are the subject of a large literature; Hong and Wang (2000) give a concise review. Price-limit "hits" can be thought of as premature market closes *though for individual stocks*, and some price-limit effects are analogous to those of market closes. For the most part, however, market closes differ substantially from daily price limits on individual stocks. Institutionally, before a stock's price hits its limit, the price is capped; prices prior to a market close are not. Closes occur at long-scheduled times and affect all stocks equally; the limit on an individual stock is hit at an unscheduled time and directly affects only that stock. A market close affects a portfolio's entire equity component from this market, but a price limit hit affects only a small part of a diversified trader's portfolio. Theoretically, after a price limit hit, an equity portfolio manager finds it more difficult to rebalance the (perhaps limited) parts of the portfolio where the stock is important; after the market close, the manager finds it much more difficult to rebalance the portfolio or change overall equity exposure in that market or (for a major market) in general. Empirically, market closes tend to generate a rush of volume; in contrast, as shown later, prices in the neighborhoods of their limits tend to have lower than average activity.

Market-wide trading halts ("circuit breakers") differ substantially from daily price limits on individual stocks. Subrahmanyam (1994) shows that traders may make sub-optimal trades in anticipation of a circuit breaker; Greenwald and Stein (1991), however, argue circuit breakers may mitigate transactional risk. The differences between price limits on individual stocks and circuit breakers have some analogies to the differences between price limits and market closes. Institutionally, before a stock's price hits its limit, the price is capped; individual prices prior to a circuit breaker are not capped in the same way. Circuit breakers affect all stocks equally; the limit on an individual stock directly affects only that stock. A circuit breaker affects a portfolio's entire equity component from this market, but a price limit hit affects only a small part of a diversified trader's portfolio. Theoretically, after a price limit hit, an equity portfolio manager finds it more difficult to rebalance the (perhaps limited) parts of the portfolio where the stock is important; with a circuit breaker, the manager finds it much more difficult to rebalance the portfolio or change overall equity exposure in that market or (for a major market) in general. Empirically, as prices approach the circuit breaker trigger, a rush of volume tends to occur (Subrahmanyam (1994)); in contrast, as shown later, individual prices in the neighborhoods of their limits tend to have lower than average activity.

Similar to price limits, trading halts (or suspensions) often apply to individual securities (Christie, Corwin, and Harris (2000) give a useful literature review). Individual trading halts differ from price limits in significant ways, however. First, unlike price limits, prices before a halt are not capped. Second, unlike price limits, trading halts are not mechanically or predictably imposed, but are subjectively imposed under certain circumstances. For example, in the NYSE, trading halts occur for two main reasons: the firm announces impending news or the market maker observes a severe order imbalance (Bhattacharya and Spiegel (1998)). Lee, Ready, and Seguin (1994) find higher volume and volatility following the halts; Ferris, Kumar, and Wolfe (1992) find similar results for a sample of SEC-ordered suspensions, and Christie et al. (2000) for Nasdaq news-related halts. Lee et al. (1994) argue that trading halts fail to reduce excessive volatility and that it would be better to have uninterrupted trading; Christie et al. (2000) reach a similar conclusion. Because of the significant differences between price limits and trading halts, however, the trading-halt literature offers only limited insight into how price limits affect markets and participants.

Kodres and O'Brien (1994) model how individual assets' price limits create time outs that benefit traders by letting them become more informed during times of uncertainty (Greenwald and Stein (1991) derive similar results for circuit breakers). Recent empirical work, however, does not support the view that individual assets' price limits aid information dissemination during times of uncertainty (e.g., Chen (1998) and Kim and Rhee (1997)). This paper's theoretical and empirical results cast further doubt on the desirability of price limits.

Because our tests use TSE data that include only open, close, high, low and limit prices, the model focuses on the informed investor's decision to trade today, the next trading day ("tomorrow"), or both days, but does not deal explicitly with the sequence of intra-day trades. In contrast, with some models in the literature the focus is on the intra-day sequence of trades, with no "today-versus-tomorrow" distinction (e.g., Easley and O'Hara (1992)).

Our informed investor's superior information is revealed to the market in the two ways that are standard in the literature. First, information leaks to the market over time even if the informed investor does nothing; in our model, which analyzes today's trades versus tomorrow's, some information leaks between today's close and tomorrow's open, and leaks completely after tomorrow's close. Second, the informed investor's trades—both today's and tomorrow's—reveal some or all of her superior information. Subject to these constraints, the informed investor wants to identify a trading strategy that will maximize her profits.

Our model analyzes cases where the price limit is binding on the informed investor. The current price is P_0; the investor receives information and estimates the new equilibrium price, P_{equil}; but the equilibrium price is beyond the current price limit, P_{limit}. On the upside, $P_0 < P_{limit} < P_{equil}$; on the downside, $P_{equil} < P_{limit} < P_0$.

Taking the upside case as an example, the informed investor has three second-best alternatives: (1) She may make all of her trades today, (2) she may begin her trading plan now and continue it tomorrow, or (3) she may put off her plan until tomorrow. Her optimal program now depends on how close P_0 is to P_{limit}, and how close P_{limit} is to P_{equil}. Intuitively, if the informed investor acquires information when (P_{limit} - P_0) is small and (P_{equil} - P_{limit}) is large, she may delay her program until tomorrow rather than reveal some of her information today for little personal gain. As this example illustrates, price limits may delay the informed investor's trades and thus impose social costs by delaying when the market receives her information.

Section 2 shows how price limits affect the investor's optimal trading plan, and discusses the model's empirical implications. A simple reduced-form model embodies results in the literature, and focuses on the choice between trading today versus tomorrow. Section 3 discusses the Taiwan Stock Exchange and the data. There are two main sets of empirical tests: Section 4 discusses the results for one set of tests, and Section 5 for the other set. Section 6 offers a summary and some conclusions.

2. The Effects of Intra-Day Price Limits

This section's model embodies a number of well-known results in the literature. First, the informed investor seeks to maximize her trading profits by choosing an optimal program. Suppose that on any given day, the set of potentially informed investors is large, but at most one investor is informed.[1] The informed investor chooses a trading plan where prices only gradually incorporate her information (Kyle (1985) and Foster and Viswanathan (1994)); her initial trades do not immediately reveal all her information because the market contains many noise traders,

[1] Kyle (1985) also assumes a single informed trader. Foster and Viswanathan (1994) extend Kyle's model to allow two informed traders with different degrees of information; Easley and O'Hara (1987) allow numerous competing informed traders. In the present model, the issue is the informed investor's behavior when she has some control over

consistent with the assumptions in Kyle (1985). In general in asymmetric information models, the market learns the informed investor's information only gradually, not instantly (Dufour and Engle (2000)).

Second, in Kyle (1985), the informed investor infers or estimates P_{equil}, given P_0, and chooses the size of her trade (X, in present notation). Easley and O'Hara (1987) point out that large trade sizes may reveal too much information; consequently, informed investors may break up their trades, suggesting that an optimal number of trades, N, exists.[2] Further, there is substantial evidence that price formation depends on observed trading behavior, or is an outcome of X and N. Jones et al. (1994) provide evidence that the number of trades reveals information for price formation (see Glosten and Milgrom (1985) for a theoretical treatment); Hasbrouck (1991) and Easley et al. (1997a,b) provide evidence that trade size affects price formation.

Third, her continued trading eventually reveals enough information to eliminate her profit opportunities (Foster and Viswanathan (1993)); at the same time, her trading drives price to its new equilibrium level.

These well-known results are embodied in two reduced forms, the profit function and the price function, to build a tractable model of how price limits affect choice of today's trades versus tomorrow's. The profit function is

$$\pi = f(X, N \mid P_0, P_{equil}).$$

Profits π depend on the size and number of trades, X and N, conditional on the current price P_0 and the equilibrium price P_{equil}. (For convenience, each of the investor's trades is the same size.) The dual of the profit function is the price function (similar to Kyle's (1985) "pricing rule"),

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the speed with which her information is revealed; considering multiple informed investors simply adds complications.

$P = P_0 + H(X, N)$.

X, N cause pricing pressure and reveal information, as discussed earlier. For $P < P_{equil}$, increases in X, N increase price, $H_X, H_N > 0$. From the profit function, the FOCs for an unconstrained interior solution are

$\partial f/\partial X = 0, \partial f/\partial N = 0$.

These yield the unconstrained optimum values X*, N*. When the informed investor carries out N* trades of size X*, she drives

$P = P_0 + H(X^*, N^*) = P_{equil}$,

where $H_X(X^*, N^*) = 0 = H_N(X^*, N^*)$.

More generally, the investor may trade both today and tomorrow. Her profit function for today is still $\pi = f(X, N)$, but her profit function for tomorrow is $\pi' = g(X', N' \mid X, N)$, where the primed variables are her choices for tomorrow, possibly different from today's. Tomorrow's profits π' are conditional on X, N, because her trading choices today reveal information. Today's size and number of trades negatively affect π', $\partial\pi'/\partial X < 0$, $\partial\pi'/\partial N < 0$; the larger X and N, the more information is revealed and the smaller tomorrow's profits. Because information leaks over time, she trades today if possible rather than tomorrow[3]; this is shown by the condition $f(x, n) > g(x, n \mid 0, 0)$ for any set $X = X' = x$, $N = N' = n$. Further, trading today is more profitable at the margin than trading tomorrow: this is shown by the condition that for $X' = 0 = N'$, and $X = x$, $N = n$, or $f(x, n)$ and $g(0, 0 \mid x, n)$, then $\partial f/\partial X > \partial g/\partial X'$, $\partial f/\partial N > \partial g/\partial N'$.

[2] Easley and O'Hara (1992) and Foster and Viswanathan (1994) consider a profit-maximizing interval between trades; Dufour and Engle (2000) find empirical support. Previous versions of this paper modeled choice of an interval, I; this complicated the analysis without changing any important result.

[3] If the market is importantly thinner or less liquid today than tomorrow, the informed investor might put off her program even if she has adequate time today. This is similar to putting off her program until a later time today when the market is thicker and more liquid.

Finally, at today's unconstrained optimum, X*, N*, there are no profitable trades tomorrow, max $\pi' = 0 = g(0, 0 \mid X^*, N^*)$, $g_{X'} < 0$, $g_{N'} < 0$.

The interesting case is where the price limit is a binding constraint: $P_{equil} > P_{limit} > P_0$. The informed investor has three second-best alternatives: (1) She completes her trading program today. (2) She starts her program today, trading until $P = P_{limit}$, and finishes tomorrow. (3) She starts and finishes her program tomorrow, when the price limit is higher and is assumed non-binding for simplicity.[4]

The informed investor's constraint is that her trades today can drive P to no more than P $= P_{limit}$, or $[P_0 + H(X, N)] \leq P_{limit}$. For the general case, the informed investor maximizes the Lagrangeian expression

$$£ = f(X, N) + g(X', N' \mid X, N) - \lambda_H [P_0 + H(X, N) - P_{limit}]$$

$$-\lambda_X X - \lambda_N N - \lambda_g g(\ldots) - \lambda_{X'} X' - \lambda_{N'} N'.$$

The FOCs are

(1) $\partial f/\partial X - \lambda_X - \lambda_H H_X + (1 - \lambda_g) \partial g/\partial X = 0,$

$\partial f/\partial N - \lambda_N - \lambda_H H_N + (1 - \lambda_g) \partial g/\partial N = 0,$

$(1 - \lambda_g) \partial g/\partial X' - \lambda_{X'} = 0,$

$(1 - \lambda_g) \partial g/\partial N' - \lambda_{N'} = 0,$

$P_{limit} \geq (H + P_0)$, $\lambda_H [P_0 + H(X, N) - P_{limit}] = 0,$

$g(\ldots) \geq 0$, $\lambda_g g = 0,$

$X \geq 0$, $\lambda_X X = 0$, $N \geq 0$, $\lambda_N N = 0,$

$X' \geq 0$, $\lambda_{X'} X' = 0$, $N' \geq 0$, $\lambda_{N'} N' = 0.$

[4] For simplicity, assume tomorrow's price limit exceeds the new rational expectations price. Of course, it might take several days of limit moves for price to rise to its new equilibrium value.

Consider the intuition of the three second-best alternatives. (The Appendix gives a more detailed discussion.)

All Trades Today. Suppose overall profit opportunities are given, in the sense that (P_{equil} - P_0) is fixed. A number of considerations may lead the investor to make all her trades today. Ceteris paribus, the informed investor is more likely to do all trading today the closer is P_{limit} to P_{equil} in $P_0 < P_{limit} < P_{equil}$. Further, the investor will make all trades today if so much information leaks overnight that, even without any trades today, it is not profitable to trade tomorrow. Finally, the investor makes all trades today if, though it may be profitable to trade tomorrow, it is more profitable to make as much as possible today rather than (a) to hold off on all trades until tomorrow, or (b) to forego some profits today to preserve profits tomorrow. In this case of all trades today, the investor makes smaller and fewer trades than if the price limit were not binding.

All Trades Tomorrow. The informed investor carries out her optimal trading program tomorrow, by assumption without a price limit constraint. Because some information leaks overnight, the program generates smaller profits than she could make today if the price limit were not binding. A combination of reasons predisposes the investor to wait until tomorrow. First, if (P_{limit} - P_0) is small but (P_{equil} - P_{limit}) is large, then trading today generates little profit because it must stop after a small price rise. Second, if trading today reveals so much information that trading tomorrow is not worthwhile, then trading today has high cost. Third, the amount of information that leaks overnight is not excessive.

Trades Both Today and Tomorrow. As compared to the case where all trading takes place tomorrow, the optimum values of tomorrow's X' and N' are smaller when some trading is done today, because today's trading reveals information that reduces tomorrow's profits: the fact that X, N > 0 rather than X = N = 0 reduces tomorrow's total and marginal profits. Trading on both days is more likely if the profits from today's trades are large, but the information leakage overnight is small and the information revealed by today's trades is small.

2.1 Empirical Implications

Hold constant the overall profit opportunities as measured by (P_{equil} - P_0), and consider the effect of where P_{limit} is located in $P_0 < P_{limit} < P_{equil}$. On the one hand, the nearer is P_{limit} to P_{equil}, and the larger is the gap between P_0 and P_{limit}, the greater is the informed investor's two-day profit by starting her program today. On the other hand, given (P_{equil} - P_0), price limits deter the informed investor's trades most when (P_{limit} - P_0) is small; ceteris paribus, the smaller is (P_{limit} - P_0), the larger is her incentive to start her program tomorrow.

Empirical tests require creativity: of the model's variables, P_0, P_{limit}, P_{equil}, only P_{limit} is observable. For this paper's sample, a stock's daily price limit is seven percent of the previous day's close, or $P_{limit,t} = (1 \pm .07)\ P_{C,t-1}$. Testable hypotheses are developed in two ways. First, one set of tests uses day t's opening price, $P_{O,t}$, as a proxy for P_0 in predictions of price changes from opening to the closing price, $P_{C,t}$. The model predicts that when ($P_{limit,t}$ - $P_{O,t}$) = ($P_{limit,t}$ - P_0) is small, ceteris paribus price is less likely to move to P_{limit} than if ($P_{limit,t}$ - $P_{O,t}$) = ($P_{limit,t}$ - P_0) is larger. The data are consistent with this prediction. This result is consistent with the idea that information-based trading is postponed until tomorrow: With no informed investors, or if informed investors always traded immediately on their information, prices opening near their limits would be more, not less, likely to hit their limits that day than those far away.

Second, another set of tests uses today's closing price, $P_{C,t}$, as a proxy for P_0 in predictions of price changes from today's closing price to tomorrow's opening price, $P_{O,t+1}$. The model predicts that when ($P_{limit,t}$ - $P_{C,t}$) = ($P_{limit,t}$ - P_0) > 0 but small, price is more likely to increase at the open and by a larger amount than if ($P_{limit,t}$ - $P_{C,t}$) = ($P_{limit,t}$ - P_0) is larger. The data are consistent with this prediction. This result is consistent with the idea that information-based trading is postponed until tomorrow: If there were no informed investors, or if informed investors always traded immediately on their information, prices near the limit would be no more likely to jump at tomorrow's open than those far away.

3. The Taiwan Stock Exchange and the Data

This paper tests our model's predictions on Taiwan Stock Exchange (TSE) data. We examine TSE data because its price-limit system is ongoing and regularly comes into effect. Further, the TSE is a useful market to study in and of itself. Based on annual turnover, the TSE is the busiest stock exchange in the world and, in recent years, its market capitalization has consistently been among the top 15 in the world.

Data are for 1991 to 1994, which is retrieved from the PACAP Database-Taiwan. In this database, daily opening, closing, high, and low prices are available for the TSE. We adjust price data to reflect capital distributions, including stock splits, reduction of capital, rights offerings, and stock dividends.

At the end of 1994, the TSE had 336 stocks listed and a market capitalization of NT\$6.70x10^{15} (US\$1=NT\$26.39 on December 30, 1994). Trading on the TSE begins at 9:00 a.m. and ends at 12:00 noon, Monday through Friday. On Saturday, trading takes place from 9:00 a.m. to 11:00 a.m. The TSE is completely order-driven and has no market makers. Orders can be entered a half-hour prior to the market open and the opening price is chosen to maximize trading volume. During most of our sample period, 1991-1994, all trades were carried out by the TSE computer-assisted trading system (CATS), which was modeled after the Toronto and Tokyo trading systems. After the morning call market determined the opening price, trading took place under a continuous auction market. At the end of the trading day, however, a call market determined closing prices. On August 2, 1993, the TSE went to a fully automated securities trading system (FAST), where trading now takes place under a call market method at 60-90 second intervals throughout the entire day. Because our results are much the same before and after this change, it appears that this change in trading method did not alter the effects of price limits.

The TSE daily price limit for each stock is set at $\pm$ 7 percent from the stock's previous closing price. The 1996 TSE Fact Book states, "In order to maintain a stable stock market, the daily price limits of stocks... are set at 7 percent of the closing price of the preceding business day." During the trading day, stocks that hit their price limit are still allowed to trade as long as the transaction price is within the limits.

4. Tests of Delays in Trading Programs

This section begins with some empirical results that suggest that testing the model may be worthwhile. It then discusses results from one of the two main sets of empirical tests.

4.1. Preliminary Empirical Results: Distortions in the Distribution of Returns

If price limits affect informed investors' trading behavior, then the distribution of price changes should reflect this, particularly when prices are close to their limits. To investigate this distribution, one-day price changes are assigned to 10 categories, following Kim and Rhee (1997). Stock prices that reach their daily price limit are classified as 'stocks_{hit}.' One-day price changes that are smaller in absolute value are categorized as $\text{stocks}_{0.90}$, $\text{stocks}_{0.80}$, $\text{stocks}_{0.70}$,….., $\text{stocks}_{0.20}$, and $\text{stocks}_{0.10}$ For example, $\text{stocks}_{0.90}$ includes stocks whose prices experienced an absolute one-day price change of at least 0.90 of the maximum permitted but *did not hit* their price-limits; therefore, $\text{stocks}_{0.90}$ experience a one-day price change of at least 6.3 percent (0.90 x

7 percent price limit), but less than the 7 percent maximum.[5] Similarly, $stocks_{0.80}$ experience a price change of at least 80 percent of the maximum but less than 90 percent, and so on. Figure 1a shows the number of observations for each stock category. Figure 1b gives a close-up view of $stocks_{hit}$, $stocks_{0.90}$, … , $stocks_{0.60}$ (sample sizes are given in the bars of the graph). Figure 2, for each year, with the price rises and declines treated separately, shows that the patterns in Figures 1a and 1b persist across years and apply to both price increases and decreases.

[Insert Figures 1a, 1b and 2 Here]

Figure 1a shows a 'spike' at $stocks_{hit}$ (far right hand side of the graph). This suggests that these stocks' equilibrium prices are at or beyond the limit-price, but the price changes are constrained by the limits. As expected, sample sizes decrease from $stocks_{0.10}$ to $stocks_{0.80}$ (the figure uses 'S' to denote 'stocks'): as the size of the one-day price change increases, the probability of its occurrence should decrease.

The sample size for $stocks_{0.90}$ is inconsistent with the pattern for $stocks_{0.10}$ to $stocks_{0.80}$. Ceteris paribus, the expected sample size for $stocks_{0.90}$ is smaller than for $stocks_{0.80}$: Instead, the sample size is 8,036 for $stocks_{0.90}$ but only 7,566 for $stocks_{0.80}$. In going from $stocks_{0.10}$ to $stocks_{0.20}$, from $stocks_{0.20}$ to $stocks_{0.30}$, etc., the average (minimum, maximum) decline is 31.7

[5] To be more precise, a stock is counted in $stocks_{hit}$ if its price is at its limit or less than one tick from its limit. TSE tick sizes vary with the stock price:

Price (P)	Tick Size
P < NT$5	NT$0.01
NT$5 ≤ P < NT$15	NT$0.05
NT$15 ≤ P < NT$50	NT$0.10
NT$50 ≤ P < NT$150	NT$0.50
NT$150 ≤ P < NT$1,000	NT$1.00
NT$1,000 ≤ P	NT$5.00

Thus, if the limit is NT$ 10, the stock is included in $stocks_{hit}$ if the price is > NT$ 9.95. The upper limit for the stock to be included in $stocks_{0.90}$ is NT$ 9.95. Similar considerations govern the other categories. In particular, no stock in $stocks_{0.90}$ is prevented by tick size from moving to $stocks_{hit}$.

percent (22.6 percent, 41.4 percent). From this, the expected sample size in $stocks_{0.90}$ might be 5,168 (± 700), which is more than one-third smaller than the observed 8,036.

The model predicts that when $P_{equil} > P_{limit}$ and (P_{limit} - P_0) is small, the informed investor may postpone trades until tomorrow. In this interpretation, some of the stocks in $stocks_{0.90}$ have equilibrium prices above today's price limits; informed investors do no drive prices to their limits, however, but delay their trades to the next day to earn larger profits.

4.2. Tests of Delays in Trading Programs

This section compares price dynamics across stocks that have different gaps between their *opening* prices and their price limits. Because every stock's price limit is ± 7 percent of the previous day's *closing* price, $P_{C,t-1}$, each stock's price limits on day t, $P_{limit,t}$, are the same ± 7% from $P_{C,t-1}$. A call market determines day t opening prices ($P_{O,t}$), and thus $P_{O,t}$ and $P_{C,t-1}$ are not necessarily equal. Immediately after the opening, the gap $P_{O,t} \pm P_{limit,t}$ differs in percentage terms across stocks, perhaps dramatically; those stocks where $P_{O,t} = P_{C,t-1}$ have a percentage gap of 7 percent, but those stocks that open at their limits have a zero percentage gap.

In terms of Section 2's model, think of $P_{O,t}$ as P_0. The model predicts that for those stocks where $P_0 < P_{limit} < P_{equil}$, the closer is P_0 to P_{limit}, the less likely is the informed investor to trade on day t, and the more likely to postpone trading until day t+1. Using $P_{O,t}$ as an operational proxy for P_0, the prediction is that the closer is $P_{O,t}$ to P_{limit}, the less likely is the informed investor to trade today, and the less likely is the stock to hit its limit today.

Stocks are categorized based on their opening prices relative to their price limits, and the percentage of stocks in each category that hit their price limit later that same day is calculated. Stocks whose opening prices differ from their closing prices by at least 90 percent of the amount permitted by the price limits, but *do not* open at their price limits, are in the category $OPENS_{0.90}$.

In other words, $OPENS_{0.90}$ contains stocks whose opening price experienced a price change from the previous day's close by at least 6.3 percent (= 0.90 x 7%), but less than the 7 percent price limit. This group contains stocks that opened closest to their price limits. $OPENS_{0.80}$ contains stocks whose opening prices differ from their previous day's closing prices by at least 80 percent of their limits but by less than 90 percent, with $OPENS_{0.70}$, $OPENS_{0.60}$, $OPENS_{0.50}$, $OPENS_{0.40}$, and $OPENS_{0.30}$ similarly defined.[6]

Table 1 shows results for stocks that experienced $close_{t-1}$-to-$open_t$ price increases or decreases. For each stock category, Table 1 shows the sample size (n), and also the percentage of stocks that hit price limits later that day (% hit). For example, the sample size for $OPENS_{0.30}$ is 7,170 for price increases and 3,318 for price decreases; in other words, there were 7,170 cases where the close-to-open price increase was at least 2.1 percent (0.30 x 7% price limit) but was less than 2.8 percent (0.40 x 7% price limit). For $OPENS_{0.30}$, less than 13 percent of the stocks hit their price limits later that day.

[Insert Table 1 Here]

In Table 1, in going from $OPENS_{0.30}$ to $OPENS_{0.80}$, sample sizes decrease with increases in the size of the price changes, as expected: As the size of the price change increases, the probability of observing the change is expected to decrease. Again as expected, the percentage of stocks that subsequently hit their price limits increases monotonically in going from $OPENS_{0.30}$ to $OPENS_{0.80}$. Suppose that, after the opening, the distribution of subsequent equilibrium percentage changes in price is roughly the same across the categories. Then, the larger is the opening price, and hence the smaller the gap between the opening price and the price

[6] Because stocks in $OPENS_{0.20}$ and $OPENS_{0.10}$ rarely hit their price limits, examining these groups yields little additional information.

limit, the more likely is the subsequent equilibrium price to be beyond the price limit and the more likely is the stock to hit its price limit.

For evaluating the hypothesis that price limits affect the informed investor's trading strategies, $OPENS_{0.90}$ is the crucial category. Table 1 provides two pieces of evidence. First, the sample size for $OPENS_{0.90}$ is larger than for $OPENS_{0.80}$ for both price increases and decreases. This is unexpected. The model provides an interpretation. $OPENS_{0.90}$ contains some stocks that have equilibrium prices beyond their day t limits. Rather than driving these stocks' prices to their limits, informed investors hold off until tomorrow, day t+1.

Second, after the day t opening, a smaller percentage of stocks in $OPENS_{0.90}$ hit their price limits later that day than do stocks in $OPENS_{0.80}$, though $OPENS_{0.90}$ contains the opening prices closest to their limits. Approximately 31 percent of stocks in $OPENS_{0.90}$ hit their price limits later that day—31.8% for price increases, 30.8% for decreases. For both the price-increase and -decrease samples, percentages for $OPENS_{0.80}$, $OPENS_{0.70}$, and $OPENS_{0.60}$ are larger than for $OPENS_{0.90}$.

In the absence of informed traders and their behavior, the stocks in $OPENS_{0.90}$ are expected to have the highest probability of hitting price limits later that day, because their opening prices are closest to their price limits. The unexpectedly small percentage of stocks in $OPENS_{0.90}$ that hit their price limits later today is consistent with the view that informed investors are waiting until tomorrow to begin their trading programs in these stocks, as the model suggests. The model further suggests that for stocks with opening prices farther from their price limits, informed investors are more likely to start their trading programs today and drive the price to $P = P_{limit}$. The data are consistent with this view: for both price increases and decreases,

$OPENS_{0.80}$, $OPENS_{0.70}$ and $OPENS_{0.60}$ all have larger percentages of stocks that hit their limits later that day than does $OPENS_{0.90}$.

The results for $OPENS_{0.60}$ – $OPENS_{0.80}$ are consistent with the model's prediction that the informed investor is more likely to begin her trading program today if price is *not* near its limit, even if the equilibrium price is beyond the limit, $P_{O,t} < P_{limit,t} < P_{equil}$. The results for $OPENS_{0.90}$ are consistent with the model's prediction that the informed investor is less likely to begin her trading program today, and more likely to postpone it until tomorrow, if price *is near* its price limit.

5. Tests of Price Continuations and Price Reversals

This section presents the second set of test results. It uses day t closing prices, $P_{C,t}$, and day t+1opening prices, $P_{O,t+1}$, to test the model's predictions. The tests focus primarily on stocks where day t closing prices are also day t highs or lows. For such stocks at daily highs, closing-to-opening price rises are price continuations, and price declines are price reversals, with definitions similar for stocks that close at daily lows, mutatis mutandis. Tests are formulated for the mean values of continuations and reversals, and the percentages of each. The section begins with some preliminary evidence that suggests that these tests are worthwhile.

5.1. Preliminary Empirical Evidence

For the categories, $stocks_{hit}$, …, $stocks_{0.60}$, Figure 3a shows the percentage of times that the stock's closing price equals its day t high, Figure 3b its low, price. If closing prices are equilibrium prices, P_{equil}, then these percentages should be essentially uniform save for $stocks_{hit}$, where limits constrain some prices to differ from their equilibrium values. In Figure 3, $stocks_{0.60}$, $stocks_{0.70}$, and $stocks_{0.80}$ close at their daily high or low price around 25 percent of the time.

In contrast, for one-day price *increases*, 37 percent of $stocks_{0.90}$ close at their high; for *decreases*, 30 percent of $stocks_{0.90}$ close at their low. These results for $stocks_{0.90}$ are consistent with the model. The model predicts that if price is close to its limit, and the equilibrium price is beyond the limit, $P_0 < P_{limit,t} < P_{equil}$, then informed investor is likely to hold off on her trading program until tomorrow, rather then drive the price to its limit today.

[Insert Figure 3 Here]

More important, the model predicts that the informed investor will begin her program tomorrow, and consequently stocks in $stocks_{0.90}$ will show more and larger price changes at tomorrow's opening than stocks with closing prices farther from their limits. Put another way, tomorrow's opening prices are expected to show more pronounced *price continuation* behavior for stocks that closed nearer their limits.

5.2. Price Continuations and Price Reversals

This sub-section presents tests that are designed to isolate the effects of revisions in price limits from day t to day t+1. These tests focus on stocks where day t closing prices are day t highs or lows,[7] and examine continuations and reversals by comparing the opening $P_{O,t+1}$ with the immediately previous price, the closing $P_{C,t}$. Movements in these *consecutive* or *adjacent* prices should reveal evidence of the effects of price-limit revisions. The model predicts price continuation for stocks that are near but not at their price limits. Across stock categories, cross-sectional mean close-to-open returns are reported, and the number of price continuations and price reversals are also analyzed. In the following sub-section, regressions on closing-to-closing returns are used to control for the effects of market movements on continuations and reversals.

Close-to-open returns are calculated as $\ln(P_{O,t+1}/P_{C,t})$, where $P_{O,t+1}$ is the opening price on day t+1 (tomorrow), $P_{C,t}$ the closing price on day t (today), and ln the natural logarithm operator.

Mean Close-to-Open Returns. Column (1) in Table 2 reports the cross-sectional mean $close_t$-to-$open_{t+1}$ returns for $stocks_{hit}$, $stocks_{0.90}$, …, $stocks_{0.60}$, for both day t price increases and day t decreases. Significance levels found from t-tests are reported for $close_t$-to-$open_{t+1}$ mean returns; unless otherwise noted, *all* reported significance levels are the same in a sign test and the Wilcoxon signed-rank test. Column (5) reports sample size.

[Insert Table 2 Here]

Table 2, Panel A reports the mean $close_t$-to-$open_{t+1}$ returns for stock categories with day t price increases. From column (1), all stock groups experience overall positive mean returns, $\ln(P_{O,t+1}/P_{C,t}) > 0$, or show price continuation on day t+1. $Stocks_{hit}$ shows the largest price continuation, with a mean return of 2.10%; this suggests that price limits restrain efficient price discovery, consistent with findings by Chen (1998) and Kim and Rhee (1997). Table 2, Panel B, shows similar results for stock categories with day t price decreases.

More important are the results for $stocks_{0.90}$. In Panel A, among the *non-hit* stock groups, $stocks_{0.90}$ shows the largest continued price increases, with a mean return of 1.24%; this mean is statistically significantly greater than the means of the other non-hit categories, but the other categories' means are insignificantly different among themselves. In Panel B, for day t price decreases, $stocks_{0.90}$ shows relatively large price continuation, with mean price change of -1.01

[7] France, Kodres, and Moser (1994) and Kuserk (1990) correctly point out that when stocks' closing prices do not equal the day's high or low price, then an intraday 'reversal' has occurred. Price limits are designed to reduce or preclude price continuation, which is the opposite of reversal. Thus, price limits distort the distribution of what can be observed in samples in studies that examine day-to-day price movements. In addition, focusing on stocks that close at their high or low price eliminates intraday price reversals that are not associated with revision of the price limit.

percent; this mean is statistically significantly greater than the means of the other non-hit categories.[8]

The results for $stocks_{0.90}$ in Table 2, Panels A and B, are consistent with the model. It appears that on day t, some informed investors know that $P_{C,t} < P_{limit,t} < P_{equil}$, and refrain from trading today in order to make larger profits tomorrow.

Results in Table 2 may usefully be compared to prior studies on one-day price changes that find that stock-price movements are often followed by *reversals* (for example, Brown, Harlow and Tinic (1988, 1993), Bremer and Sweeney (1991), and Bremer, Hiraki and Sweeney (1997)). Further, these prior studies find that the larger the initial price change, the larger is the size of the reversal. The finding of large *continuations* for $stocks_{0.90}$, therefore, provides compelling evidence that, when $P_{equil} > P_{limit}$ and $P_{C,t}$ is close to $P_{limit,t}$, then information-based trading is postponed to tomorrow.

Percentages of Price Continuations and Reversals. As an alternative to examining mean returns, Table 2, column (2), reports the percentage of times that the stocks in each group experience close-to-open price continuations; this approach mitigates the influence of outliers. For comparison, column (3) reports the percentage of times that price reversals occur within each group, and column (4) the percentage of times that stock prices remain the same. In Panel A, for stocks that experience price *increases* on day t, if $P_{O,t+1} > P_{C,t}$, then this is classified as a price continuation; if $P_{O,t+1} < (=) P_{C,t}$, then this is classified as a reversal (no change). In Panel B, for stocks that experience price *decreases* on day t, if $P_{O,t+1} < (>, =) P_{C,t}$, then this is classified as a price continuation (reversal, no change).

[8] In a t-test, the mean return in $stocks_{80}$ differs significantly from zero, but in the sign test the mean return is not statistically significantly different from zero. In both tests, the mean returns in $stocks_{70}$ and $stocks_{60}$ do not differ from zero.

From Table 2, Panel A, the continuation percentages in column (2) indicate that $stocks_{hit}$ and $stocks_{0.90}$ experience significantly different price continuation as compared to the other three groups,[9] consistent with cross-sectional mean return results. Price continuations occur 73.5 percent of the time for the $stock_{hit}$ group and 60.2 percent of the time for the $stocks_{0.90}$ group, both of which are significantly larger than the 48 percent-average for the other non-hit stock groups. The larger percentage of continuations for $stocks_{hit}$ is not surprising. The fact that the $stocks_{0.90}$ group experiences continuations at tomorrow's open more often than stocks whose closing prices are further away from today's limit price is consistent with model predictions.

In Panel B, for day t price decreases, continuations occur 47.7 percent of the time for the $stocks_{0.90}$ group. Among the other stock groups that did not hit price limits, continuations only occurs around 30-33 percent of the time. In fact, the continuation behavior diminishes with decreases in the magnitude of the one-day price change in day t.

5.3 Regression Results

Overall market movements may explain some of the results discussed above. To control for market movements, cross-section stock returns are regressed on explanatory variables that include a measure of the market return.

The database from which the individual stock data are drawn does not include a close-to-open market rate of return, but only close-to-close measures of market rates of return. For this reason, the cross-sectional OLS regression below uses *close-to-close* daily rates of return on individual stocks and the market to investigate price continuations and reversals:

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[9] We use a basic binomial z-score to test for statistical differences between percentages (see Kim and Rhee (1997)). For example, to test to see if the $stocks_{0.90}$ sample experiences more price continuation than the $stocks_{0.80}$ sample, we calculate the following: $z = (CON_{0.90} - PrCON_{0.80}N_{0.90}) / (PrCON_{0.80}(1-PrCON_{0.80})N_{0.90})^{0.5}$, where $CON_{0.90}$ denotes the number of price continuations that $stocks_{0.90}$ experience, $PrCON_{0.80}$ represents the proportion of price continuations that occur for the $stocks_{0.80}$ sample and is calculated as $CON_{0.80}/N_{0.80}$, where $CON_{0.80}$ denotes the number of price continuations that $stocks_{0.80}$ experience and $N_{0.80}$ represents the $stocks_{0.80}$ sample size; and $N_{0.90}$

$$R_{j,t+1} = a + b\, R_{M,t+1} + c\, R_{j,t} + d_j.$$

$R_{j,t}$ is the day t (today's) rate of return of stock j, $R_{j,t+1}$ is the following day's rate of return (tomorrow) and $R_{M,t+1}$ is the PACAP equally-weighted daily market rate of return—a PACAP value-weighted index gives nearly identical results.

A significant and positive coefficient on the R_t parameter indicates price continuation, a negative coefficient indicates price reversal. Under some simple versions of the efficient market hypothesis, the coefficient on R_t should not be significantly different from zero, for example, if the risk premium is time constant. Roll (1984) argues, however, that negative first-order serial correlation in price changes is to be expected from bid-ask bounce in the absence of predictable price changes; this suggests that the coefficient on R_t may be biased downwards.

Table 3 reports the regression results, Panel A for the price increase sample, Panel B for the price decrease sample. From Table 3, market movements explain a minor part of subsequent price movements for the stock groups. The most important result, found in both panels, is that $stocks_{0.90}$ show significant price continuation; the coefficient on R_t is positive and significant, consistent with findings above. To a lesser degree, $stocks_{0.80}$ also show continuation, but the coefficient on R_t is larger for $stocks_{0.90}$ in both panels.

[Insert Table 3 Here]

These findings are impressive because the data are biased against finding them. First, close-to-close daily rates of return are used in this regression model, because all PACAP Database market indices are constructed from close-to-close daily returns. Compared to close-to-open returns, $\ln(P_{O,t+1}/P_{C,t})$, the close-to-close returns, $\ln(P_{C,t+1}/P_{C,t})$, contain noise from events that happen after the open on day t+1 and are thus less sensitive to the phenomena being

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represents the $stocks_{0.90}$ sample size. This z-score is distributed normally because sample sizes are all sufficiently large (see Olkin, Gleser, and Derman (1980, pp. 244-253)).

investigated than are close-to-open prices. Second, the parameter coefficient on R_t, according to Roll's (1984) bid-ask bounce argument, is biased downwards. Overall, Table 3's results support the view that the informed trader waits until tomorrow to execute her trading program when closing prices are near price limits today.

6. Conclusion

Many, perhaps the majority, of equities markets have limits on daily price movements. Little is known about the benefits and costs of these limits. As a start on investigating benefits and costs, this paper focuses on the consequences of limits for price revelation and thus resource allocation, especially through the effects of price limits on the behavior of informed investors. Because the model's predictions are tested on data from the Taiwan Stock Exchange (TSE), where only daily open, close, high, low and limit prices are available, the model focuses on the decision to begin trading today or the next trading day. The model shows how price limits may induce an informed investor to shift part or all of her profit-motivated trades until the next day, thus retarding the spread of information. The model predicts that these delays are particularly likely if the current price is near, but the equilibrium price is substantially beyond, today's limit.

In a series of tests on TSE data, results are consistent with the model. Results support the view both that informed investors' trades play a major role in price revelation and that price limits importantly delay price revelation.

If the equilibrium price is beyond today's limit, the informed investor faces three second-best strategies: (1) trade only today, (2) trade today and tomorrow, or (3) trade only tomorrow. The informed investor's strategy depends in large part on the distance from the current price to the limit price. If the current price is far from the price limit, then the model predicts that the investor initiates her trading program today, especially if the equilibrium price is only a small amount beyond the price limit. In contrast, if the current price is close to the price limit, and the equilibrium price is substantially beyond the limit, the model predicts that the trader may delay her trading activities to tomorrow, forgoing profit opportunities today. This case reveals a cost of price limits: the market receives information late, and current prices are non-equilibrium prices, thus distorting resource allocation. The same costs hold, but to a smaller extent, in the case where the investor starts her trading today but does not finish it until tomorrow.

A series of empirical tests support the model's predictions. The tests use data from the Taiwan Stock Exchange, which systematically uses price limits. One set of tests focuses on opening prices and compares them with subsequent price on the same day. Opening prices that are near but not at that day's limit show an unexpected and significant tendency not to hit price limits later in the day. Instead, stocks with opening prices that are farther from their limits are more likely to hit their limits later in the day. These patterns are consistent with the view that informed investors are likely to put off their trading programs if the current price—here, the opening price—is near the price limit.

Another set of tests uses closing prices on one day compared to opening prices the next day. An abnormal number of stocks have closing prices that are near but not at the price limit. These stocks' prices tend to jump at the next day's opening call auction by an unexpected and significant amount. This is consistent with the view that often informed investors have superior information but put off their trading programs to the next day when today's price—here, the closing price—is close to today's limit.

This test is supplemented by a regression test that relates close-to-close rates of return for day t+1 to rates from day t, but adjusts for market movements. Stocks that have closing prices that are near but not at their limits on day t have an unexpected and significantly positive relationship to rates of return on these stocks on day t+1.

This paper's model and empirical results support the view that price limits importantly affect the informed investor's trades and through this channel importantly affect information revelation and thus resource allocation. These findings suggest that price limits deserve further examination and a serious policy debate.

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Table 1
Opening Prices and Subsequent Price-Limit-Hits

This table presents sample sizes (n) for the following stock groups: $OPENS_{0.30}$, $OPENS_{0.40}$, $OPENS_{0.50}$, $OPENS_{0.60}$ $OPENS_{0.70}$, $OPENS_{0.80}$, and $OPENS_{0.90}$. $OPENS_{0.90}$ denote stocks whose opening prices are different from their previous day's closing price by at least 0.90($LIMIT_t$), but did not reach a daily price limit; where $LIMIT_t$ denotes the maximum allowable daily price movement on day t. $OPENS_{0.80}$ denote stocks whose opening prices are different from their previous day's closing price by at least 0.80($LIMIT_t$), but less than 0.90($LIMIT_t$), on day t. $OPENS_{0.70}$ to $OPENS_{0.30}$ are identified in a similar manner. For each stock category, we calculate the number of times that a subsequent price-limit-hit occurs on the same day t, which is reported as a percentage (%-hit).

Price Increases at the Open			**Price Decreases at the Open**		
Stock Category	n	%-hit	Stock Category	n	%-hit
$OPENS_{0.30}$	7,170	12.7%	$OPENS_{0.30}$	3,318	12.2%
$OPENS_{0.40}$	3,259	20.9%	$OPENS_{0.40}$	1,775	19.2%
$OPENS_{0.50}$	1,664	28.8%	$OPENS_{0.50}$	1,067	26.6%
$OPENS_{0.60}$	964	33.1%	$OPENS_{0.60}$	736	33.2%
$OPENS_{0.70}$	668	34.3%	$OPENS_{0.70}$	566	34.3%
$OPENS_{0.80}$	587	40.0%	$OPENS_{0.80}$	533	37.3%
$OPENS_{0.90}$	748	31.8%	$OPENS_{0.90}$	772	30.8%

Table 2
Overnight Price Continuations and Reversals

For stocks that are grouped by one-day price changes, this table provides the subsequent overnight returns. The reported returns are cross-sectional, close-to-open, mean returns from daily data on the Taiwan Stock Exchange. Close-to-open returns are calculated as $\ln(P_{O,t+1}/P_{C,t})$, where P_C is the closing price, P_O is the next day's opening price, t denotes the day the stock groups are formed, and ln denotes the natural log operator. Mean returns are calculated for the categories: $Stocks_{hit}$, $Stocks_{0.90}$, $Stocks_{0.80}$, $Stocks_{0.70}$, and $Stocks_{0.60}$. $Stocks_{hit}$ includes those stocks that hit their upper or lower price limit on day t. $Stocks_{0.90}$ denote stocks that experience a price change of at least $0.90(LIMIT_t)$ on day t, but do not reach a daily price limit; where $LIMIT_t$ denotes the maximum allowable price movement on day t. $Stocks_{0.80}$ denote stocks that experience a price change between $0.80(LIMIT_t)$ and $0.90(LIMIT_t)$ on day t. $Stocks_{0.70}$ and $Stocks_{0.60}$ are identified in a similar manner. All of the stocks in these groups have day-t closing prices that are equal to their daily high or low price. In other words, where $P_{H,t} = P_{C,t,}$ or $P_{L,t} = P_{C,t}$, where P_H and P_L represent the daily high or low price. Mean returns are presented in column (1). We also report the percentage of times price continuations, price reversals, and no change in price occurs for each of our stock groups. If a stock experiences a price increase on day t, and if $P_{O,t+1} > P_{C,t}$, then this is considered a price continuation. If $P_{O,t+1}$ < or = $P_{C,t}$, then this is considered a price reversal, or no-change, respectively. For stocks that experience a day-t price decrease, the opposite is true. Continuation (Cont), reversal (Rev), and no-change (NoΔ) percentages are presented in columns (2), (3) and (4). n denotes the sample size of each stock group. Panel A presents upper price movement results. Panel B presents lower price movement results. ** and * denote statistical significance at the 0.01 and 0.05 level, respectively.

Panel A: Price Increases Sample

Stock Category	Mean Return (1)	% Cont. (2)	% Rev. (3)	% No D (4)	n (5)
$Stocks_{hit}$	0.0210**	73.5%	13.0%	13.5%	7,653
$Stocks_{0.90}$	0.0124**	60.2%	19.6%	20.2%	1,716
$Stocks_{0.80}$	0.0033**	48.6%	25.8%	25.6%	1,171
$Stocks_{0.70}$	0.0038**	47.8%	24.2%	28.0%	1,286
$Stocks_{0.60}$	0.0033**	48.8%	21.7%	29.6%	1,845

Panel B: Price Decreases Sample

Stock Category	Mean Return (1)	% Cont. (2)	% Rev. (3)	% No D (4)	n (5)
$Stocks_{hit}$	-0.0222**	66.1%	17.9%	16.1%	4,768
$Stocks_{0.90}$	-0.0101**	47.7%	26.0%	26.3%	1,038
$Stocks_{0.80}$	-0.0028**	33.8%	29.8%	36.4%	742
$Stocks_{0.70}$	-0.0005	32.0%	31.8%	36.2%	1,097
$Stocks_{0.60}$	-0.0002	29.4%	34.3%	36.3%	1,631

Table 3
Regression Results

The following cross-sectional ordinary least squares regression is estimated to identify the relation between $R_{j,t}$ and $R_{j,t+1}$, where $R_{j,t}$ represents the daily return of each stock j on the day of their large price change. $R_{j,t+1}$ represents the following day's return.

$$R_{j,t+1} = a + b\, R_{M,t+1} + c\, R_{j,t} + d_j,$$

R_M is the equally-weighted daily market return and it represents a control variable. We run this regression for each of our stock groups: $Stocks_{hit}$, $Stocks_{0.90}$, $Stocks_{0.80}$, $Stocks_{0.70}$, and $Stocks_{0.60}$. $Stocks_{hit}$ denote those stocks that hit their upper or lower price limit on day t. $Stocks_{0.90}$ denote stocks that experience a price change of at least $0.90(LIMIT_t)$ on day t, but do not reach a daily price limit; where $LIMIT_t$ denotes the maximum allowable daily price movement on day t. $Stocks_{0.80}$ denote stocks that experience a price change between $0.80(LIMIT_t)$ and $0.90(LIMIT_t)$ on day t. $Stocks_{0.70}$ and $Stocks_{0.60}$ are identified in a similar manner. The study-samples only include those stocks that closed at their daily high (P_H) or low (P_L) price on the day of their large price change (day-t), i.e., when $P_{H,t} = P_{C,t}$ or when $P_{L,t} = P_{C,t}$, where P_C denotes the closing price. Panels A and B show separate results for upward and downward price changes, respectively. We also report the adjusted R^2, the F-value, and the sample size (n). ** and * denote statistical significance at the 0.01 and 0.05 levels, respectively.

Panel A: Price Increases Sample

Stock Group	Intercept	$R_{M,t+1}$	$R_{j,t}$	Adj. R^2	F-value	n
$Stocks_{hit}$	0.067**	0.007**	0.000	0.004	17.19**	7,645
$Stocks_{0.90}$	0.063**	0.008*	0.011**	0.015	13.81**	1,720
$Stocks_{0.80}$	0.055**	0.007	0.007*	0.004	3.45*	1,170
$Stocks_{0.70}$	0.048**	0.017**	0.003	0.013	9.56**	1,291
$Stocks_{0.60}$	0.041**	0.019**	0.002	0.013	12.81**	1,849

Panel B: Price Decreases Sample

Stock Group	Intercept	$R_{M,t+1}$	$R_{j,t}$	Adj. R^2	F-value	n
$Stocks_{hit}$	-0.067**	0.008**	0.002**	0.008	19.91**	4,758
$Stocks_{0.90}$	-0.063**	0.006	0.018**	0.035	19.78**	1,038
$Stocks_{0.80}$	-0.055**	0.018**	0.015**	0.046	18.81**	745
$Stocks_{0.70}$	-0.048**	0.032**	0.003	0.045	26.83**	1,096
$Stocks_{0.60}$	-0.041**	0.033**	0.002	0.034	29.58**	1,634

Appendix: The Informed Investor's Optimum Trading Program

This appendix discusses in more detail the informed investor's optimum trading program, in particular, the three possible patterns of when she trades. Section 1 discusses the profit function and the pricing function. Section 2 discusses the case where the price limit is a constraint.

1. The Profit Function, The Price Function, and Optimum Trading Strategy

Write the profit function as

$$\pi = f(X, N \mid P_0, P_{equil}).$$

Profits are conditional on the current price, P_0, and the new estimated equilibrium price, P_{equil}; the informed investor chooses X and N to maximize profits. The price function

$$P = P_0 + H(X, N)$$

is the dual of the profit function, where H_X, $H_N > 0$ for $P < P_{equil}$. The more active the informed investor's trading program, the higher it drives price. The investor makes profits as long as she buys (sells) at prices below (above) the equilibrium price, and can make no further profits when price is driven to its equilibrium value, $P = P_{equil}$. In terms of the dual price function, if $(P_0 + H) = P < P_{equil}$, further trading will produce extra profits; when $(P_0 + H) = P_{equil}$, then further trading results in zero marginal profits.

For an unconstrained interior optimum, FOCs for profits are

$$\partial f/\partial X = f_X = 0 = \partial f/\partial N = f_N = 0.$$

Denote by X*, N* the values of X, N that satisfy the FOCs in the unconstrained case. Then, $P = P_0 + H(X^*, N^*) = P_{equil}$; further trading results in zero marginal profits (before transaction costs), and has no effect on price, $H_X(X^*, N^*) = 0 = H_N(X^*, N^*)$.

The profit function implies the iso-profit curves in the X,N plane are ellipsoids around X*,N*; profits are a "mountain" with its peak at X*,N*. Because the profit function's unconstrained maximum is at X*, N*, changes in X or N from this optimum reduce profits.

The SOCs are that the matrix F is negative definite,

$$F = \begin{bmatrix} f_{XX} & f_{XN} \\ f_{NX} & f_{NN} \end{bmatrix}$$

This requires that the determinant | F | is positive and the sign of the minors of order n is sgn[$(-1)^n$]. Thus, f_{XX}, $f_{NN} < 0$, and $(f_{XX}\, f_{NN} - f_{XN}) > 0$.

Effects on The Trading Strategy of An Increase in the Equilibrium Price. In key experiments, the SOCs are not sufficient to sign results. Suppose P_{equil} increases. In matrix notation, the effects are

$$F\, [dX/dP_{equil},\, dN/dP_{equil}]' = -\, [\partial f_X/\partial P_{equil},\, \partial f_N/\partial P_{equil}]'.$$

Even if the increase in P_{equil} raises marginal profit for both X, N, or $\partial f_X/\partial P_{equil}$, $\partial f_N/\partial P_{eqyil} > 0$, further assumptions are required to sign effects. Take

$$dX/dP_{equil} = -\, (|\, F_{11} \,| \,/\, |\, F \,|)\, \partial f_X/\partial P_{equil} + (|\, F_{12} \,| \,/\, |\, F \,|)\, \partial f_N/\partial P_{equil}$$

where $F_{i,j}$ is the determinant of the matrix that results when the ith row and jth column is struck from F, for example, $|\, F_{12} \,| = f_{XN}$. From the SOCs, $|\, F_{11} \,| > 0$ and $|\, F \,| < 0$, and thus $-\, (|\, F_{11} \,| \,/\, |\, F \,|)\, \partial f_X/\partial P_{equil} > 0$. The SOCs do not determine the sign $f_{XN} = |\, F_{12} \,|$, and similarly for $|\, F_{21} \,|$. Thus, the sign of dX/dP_{equil} is indeterminate, and similarly for dN/dP_{equil}.

Intuitively, the expansion path for X, N is likely to be positive, and this is assumed in what follows. An increase in P_{equil} shifts the profit mountain: It has a higher peak and new optimum values of X, N. Ceteris paribus, attaining the new profit maximum requires more trading—either larger trades, more trades, or increases in both. The expansion path traces these optimum values as P_{equil} rises, and with a positive expansion path, the point X*,N* moves to the northeast as P_{equil} rises.[10]

Analysis of the effect of the initial price, P_0, is similar. The SOCs are insufficient to sign all of the effects. Under the assumption of a positive expansion path, however, an increase in P_0, with P_{equil} held constant, causes decreases in X and N.

The Effects of the Price Limit on Today's Trades. When the equilibrium price is beyond today's limit, the informed investor faces the situation $P_0 < P_{limit} < P_{equil}$, taking as the example the case where the equilibrium price exceeds the current price. In this situation, the informed investor's constraint is that she cannot make trades after she drives P to P_{limit}, or her price-limit constraint is

$$[P_0 + H(X, N)] \leq P_{limit}.$$

In this case, the informed investor maximizes the Lagrangeian expression

$$£ = f(X, N) - \lambda_H [P_0 + H(X, N) - P_{limit}] - \lambda_X X - \lambda_N N.$$

The FOCs are

$$\text{(A.1)} \quad \partial f/\partial X - \lambda_X - \lambda_H H_X = 0 = \partial f/\partial N - \lambda_N - \lambda_H H_N = 0,$$

$$P_{limit} \geq (H + P_0), \ \ \lambda_H [P_0 + H(X, N) - P_{limit}] = 0,$$

$$X \geq 0, \lambda_X X = 0, N \geq 0, \lambda_N N = 0.$$

For an interior maximum, X, N > 0, and thus $\lambda_X = \lambda_N = 0$. Because the price limit is a binding constraint, $[P_0 + H(X, N) - P_{limit}] = 0$ and $\lambda_H > 0$. Graphically, the concave pricing function constrains the highest iso-profit curve the informed investor can reach. An increase in P_{limit}

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[10] Suppose that the profit function is homogeneous of some positive degree α in P_{equil},X,N, or

$$\pi = f(X, N \mid P_0, P_{equil}) = (P_{equil})^{\alpha} f(X/P_{equil}, N/P_{equil} \mid P_0, 1).$$

Then, an increase in P_{equil} results in proportionate increases in X and N. The FOCs become

$$\partial f/\partial X = (P_{equil})^{\alpha - 1} f_X(X/P_{equil}, N/P_{equil} \mid P_0, 1) = f_X(X/P_{equil}, N/P_{equil} \mid P_0, 1) = 0,$$

$$\partial f/\partial N = (P_{equil})^{\alpha - 1} f_N(X/P_{equil}, N/P_{equil} \mid P_0, 1) = f_N(X/P_{equil}, N/P_{equil} \mid P_0, 1) = 0.$$

shifts the constraint out and the investor can reach a high level of profits, inducing larger values of X and N.

Think of profit opportunities as given in the sense that P_0 and P_{equil} are given, and thus (P_{equil} - P_0) is given: The profit mountain and its iso-profit curves are then given. With given profit opportunities (P_{equil} - P_0), the effect of the price limit depends on where P_{limit} falls in $P_0 < P_{limit} < P_{equil}$. Profits, X and N all increase monotonically with increases in P_{limit} as long as the price limit is a binding constraint, $P_{limit} < P_{equil}$.

Profits from Trading Today and Tomorrow. The informed investor's profit function over today and tomorrow is

$$\Pi = \pi + \pi' = f(X, N \mid P_0, P_{equil}) + g(X', N' \mid X, N; P_0, P_{equil}).$$

The properties of g(…) reflect the fact that information leaks between today and tomorrow, and that today's program [X, N] also provides information, which may interact with the tendency for information to leak. First, for any program that might be run today, simply postponing it until tomorrow results in lower profits. This is formalized as follows. For values x, n > 0,

$$f(x, n \mid P_0, P_{equil}) > g(x, n \mid 0, 0; P_0, P_{equil}).$$

Second, for a given program, marginal profits are smaller tomorrow than today. This is formalized as follows. For $f(x, n \mid P_0, P_{equil})$ and $g(0, 0 \mid x, n; P_0, P_{equil})$, the marginal profit from an increase in X today is larger than the marginal profit from an increase in X' tomorrow, $\partial\pi/\partial X > \partial\pi'/\partial X'$, and similarly N. Third, for the program that is optimal today in the absence of constraints, X*, N*, the program that maximizes profits tomorrow has no trading and gives zero profits,

$$\max g = g(0, 0 \mid X^*, N^*; P_0, P_{equil}).$$

X and N have a positive expansion path under these circumstances. Of course, homogeneity is stronger than is required to yield a positive expansion path.

In a slightly less general formulation, profits might be written as

$$\Pi = \pi + \pi' = f(X, N \mid 0, 0; P_0, P_{equil}) + \gamma f(X', N' \mid X, N; P_0, P_{equil}),$$

where g() has the same functional form as f(). The parameter $1 > \gamma \geq 0$ shows the degree to which information leaks out overnight; there is always some leakage that damages profits ($1 > \gamma$) and leakage may be great enough to destroy profits ($\gamma = 0$).

2. Price Limits As a Constraint: The General Case of Trading Over Two Days

When the price limit is a binding constraint, the informed investor considers three second-best alternatives. (1) She completes her trading program today. (2) She starts her program today, trading until $P = P_{limit}$, and finishes tomorrow. (3) She starts and finishes her program tomorrow, when the price limit is higher (and is assumed non-binding).[11] This section discusses the conditions that influence the informed investor to choose one alternative over the others. In particular, it examines the case where profit opportunities, as measured by $(P_{equil} - P_0)$, are given, and investigates the effect of where the price limit P_{limit} falls in $P_0 < P_{limit} < P_{equil}$.

For the general case, the informed investor maximizes the Lagrangeian expression

$$£ = f(X, N) + g(X', N' \mid X, N) - \lambda_H [P_0 + H(X, N) - P_{limit}]$$

$$-\lambda_X X - \lambda_N N - \lambda_g g(\ldots) - \lambda_{X'} X' - \lambda_{N'} N'.$$

The FOCs are

(A.2) $\partial f/\partial X - \lambda_X - \lambda_H H_X + (1 - \lambda_g) \partial g/\partial X = 0,$

$\partial f/\partial N - \lambda_N - \lambda_H H_N + (1 - \lambda_g) \partial g/\partial N = 0,$

$(1 - \lambda_g) \partial g/\partial X' - \lambda_{X'} = 0,$

$(1 - \lambda_g) \partial g/\partial N' - \lambda_{N'} = 0,$

$P_{limit} \geq (H + P_0), \ \lambda_H [P_0 + H(X, N) - P_{limit}] = 0,$

$g(\ldots) \geq 0,\ \lambda_g\, g = 0,$

$X \geq 0, \lambda_X\, X = 0, N \geq 0, \lambda_N\, N = 0,$

$X' \geq 0, \lambda_{X'}\, X' = 0, N' \geq 0, \lambda_{N'}\, N' = 0.$

All Trades Today, No Trades Tomorrow. With no trades tomorrow, $g = 0 = X' = N'$. $\lambda_g = 1$ so $(1 - \lambda_g) = 0$ in system (2): The investor does not trade tomorrow and hence ignores effects of today's decisions on tomorrow's, g. The informed investor trades until she drives the price today to the limit, $[P_0 + H] = P_{limit}$, so $\lambda_H > 0$. $X > 0$ and $N > 0$, so $\lambda_X = \lambda_N = 0$. The FOCs become

$\partial f/\partial X - \lambda_H H_X = 0,$

$\partial f/\partial N - \lambda_H H_N = 0,$

$[P_0 + H(X, N) - P_{limit}] = 0.$

Because $\lambda_H, H_X, H_N > 0$, the FOCs imply $\partial f/\partial X > 0$ and $\partial f/\partial N > 0$: The informed investor makes smaller trades and fewer trades than if the price limit were not a binding constraint. Denote the optimal values of today's X and N by X**, N** in the case of a binding price limit; then $X^* > X^{**}$ and $N^* > N^{**}$.

In the case of a binding price limit, an increase in P_{limit} results in increases in π, X and N under the above assumption of a positive expansion path. In particular, if P_{limit} is close to P_{equil} and far from P_0 in the inequality $P_0 < P_{limit} < P_{equil}$, then the effects of the price constraint are minimal.

This case may arise for a number of reasons, perhaps in combination. First, it may not be profitable to trade tomorrow, no matter what is done today, because of information leakage, $\gamma = 0$. Second, it may be more profitable to do all trading today, even if postponing trading allows positive profits tomorrow. At today's closing, trading pushes price to the price limit, but this is

[11] For simplicity, assume tomorrow's price limit exceeds the equilibrium price P_{equil}. Of course, in practice it may

less than the equilibrium price, $P = P_{limit} < P_{equil}$. Overnight, however, because of information leakage and because of the information revealed by today's trading program, investors recognize that the equilibrium price is P_{equil}, and price is driven to this level before the informed investor can make any profitable trades. With $\gamma > 0$, max $g = g(X', N' \mid 0, 0) > 0$; in the absence of trading today, there would be some profits to exploit tomorrow. max $g = g(0, 0 \mid X^{**}, N^{**}) = 0$ arises because today's trades have substantial effects on tomorrow's profits by revealing information. Put another way, this situation arises because $\partial g/\partial X$, $\partial g/\partial N$, $\partial^2 g/\partial X'\partial X$, $\partial^2 g/\partial X'\partial N$, $\partial^2 g/\partial N'\partial N$ are sufficiently negative.

All Trades Tomorrow, None Today. In this case, in system (A.2), $P_{limit} > [H(0, 0) + P_0] = P_0$, so $\lambda_H = 0$. The fact she trades tomorrow implies max $g = g(X', N' \mid 0, 0) > 0$, so $\lambda_g = 0$. With no trades today, but active trading tomorrow, $X = N = 0$, $\lambda_X, \lambda_N > 0$; $X', N' > 0$, $\lambda_{X'} = \lambda_{N'} = 0$. The FOCs become

$$\partial g/\partial X' = 0 = \partial g/\partial N'.$$

The informed investor carries out her optimal trading program tomorrow without constraint. Because she carries out her program tomorrow, and in general some information leaks out over night, the program generates smaller profits than what she could make today if the price limit were not a binding constraint. Denote the optimum values of X' and N' as X'* and N'*; then $f(X^*, N^*) > g(X'^*, N'^* \mid 0, 0)$. Further, $X'^* < X^*$ and $N'^* < N^*$: $\partial g(X' = X^*, N' = N^* \mid 0, 0)/\partial X' < \partial f(X^*, N^*)/\partial X = 0$, and similarly for N*.

The informed investor waits until tomorrow to trade for a combination of reasons. First, trading today generates little profit, given the fact that it must stop after a small price rise. Second, any trading today reveals so much information that trading tomorrow is not worthwhile,

require several days of limit moves for price to rise to its new equilibrium value.

or at least tomorrow's trading profits are severely impaired. Third, little information leaks out overnight (γ is less than but close to unity).

Denote by π^{**} the amount of today's trading profits in the face of a price limit constraint, associated with X** and N**. The closer is the price limit P_{limit} to the initial price P_0, the smaller is π^{**}. If the investor does not trade today, she makes optimal trading profits tomorrow of $\pi'^* = g(X'^*, N'^* \mid 0, 0)$. With a tight enough price limit today, and modest enough leakage overnight, $\pi'^* > \pi^{**}$, and doing all trading tomorrow makes sense. Further, if trades today reveal so much information that tomorrow's profits are severely affected relative to what may be earned today, $(\partial f/\partial X - \partial g/\partial X) < 0$, then trading on both days is less profitable than trading only tomorrow.

Trades Both Today and Tomorrow. Today's trades drive the price to the limit: $P_{limit} = [P_0 + H]$, so $\lambda_H > 0$. For tomorrow's trades to make sense, max g(…) > 0, so $\lambda_g = 0$. X, N > 0, so $\lambda_X = \lambda_N = 0$; X', N' > 0, so $\lambda_{X'} = \lambda_{N'} = 0$. The FOCs become

$$\partial f/\partial X - \lambda_H H_X + \partial g/\partial X = 0,$$

$$\partial f/\partial N - \lambda_H H_N + \partial g/\partial N = 0,$$

$$[P_0 + H(X, N) - P_{limit}] = 0,$$

$$\partial g/\partial X' = 0 = \partial g/\partial N'.$$

On the one hand, as compared to the case where all trading is tomorrow, the marginal conditions $\partial g/\partial X' = 0 = \partial g/\partial N'$ are satisfied at smaller values of X' and N'; the fact that X, N > 0 rather than X = N = 0 reduces tomorrow's marginal profits. Denote by X'**, N'** the values of tomorrows optimal trades when there are constrained trades today. Then, X'** < X'* and N'** < N'*.

On the other hand, the optimal size and number of today's trades are unchanged from when all trading is done today. Today's trades push price to the limit, and thus set $P = P_0 +$

$H(X^{**}, N^{**}) = P_{limit}$. The partials $\partial f/\partial X$ and $\partial f/\partial N$ are the same as in the case when there is no trading tomorrow, as are H_X and H_N. In the face of the fact that $\partial g/\partial X$, $\partial g/\partial N < 0$, the marginal conditions hold because λ_H declines, though it remains positive. λ_H measures the marginal profits foregone as a result of the price-limit constraint. To the extent that some trading tomorrow is profitable, this loss is smaller and λ_H declines to set the same value of $\partial f/\partial X$ equal to ($\lambda_H H_X$ - $\partial g/\partial X$), and similarly for N. Denote by $\lambda_H{}^{**}$ and $\lambda_H{}^{***}$ the values of λ_H when today's trades are constrained and there are no or some trades tomorrow. Then, $0 < \lambda_H{}^{***} < \lambda_H{}^{**}$.

Because the price limit P_{limit} is substantially above the initial price P_0, the profits from today's trades, $\pi^{***} = f(X^{***}, N^{***})$, are large. They are so large that—compared to the positive profits that could be made tomorrow, in the absence of trade today, max $\pi'^{*} = g(X'^{*}, N'^{*} \mid 0, 0)$—foregoing trading today in favor of trading tomorrow is not worthwhile: $\pi^{***} > \pi^{*}$. As compared to the case where there is only trading today, even when today's profits are maximized, it is still profitable to trade tomorrow: max $\pi'^{**} = g(X'^{**}, N'^{**} \mid X^{***}, N^{***}) > 0$. This case arises because there is little leakage of information overnight and because the information revealed today by X^{***} and N^{***} does not completely destroy tomorrow's potential profits. This requires a substantial γ and small partials $\partial g/\partial X$, $\partial g/\partial N < 0$.