EOQ Model

Economic Order Quantity

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EOQ Assumptions

- Known & constant demand
- Known & constant lead time
- Instantaneous receipt of material
- No quantity discounts
- Only order (setup) cost & holding cost
- No stockouts
## Inventory Holding Costs

*Reasonably Typical Profile*

<table>
<thead>
<tr>
<th>Category</th>
<th>% of Inventory Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing (building) cost</td>
<td>6%</td>
</tr>
<tr>
<td>Material handling costs</td>
<td>3%</td>
</tr>
<tr>
<td>Labor cost</td>
<td>3%</td>
</tr>
<tr>
<td>Inventory investment costs</td>
<td>11%</td>
</tr>
<tr>
<td>Pilferage, scrap, &amp; obsolescence</td>
<td>3%</td>
</tr>
<tr>
<td><strong>Total holding cost</strong></td>
<td><strong>26%</strong></td>
</tr>
</tbody>
</table>
EOQ Model
EOQ Model

Annual Cost

Order Quantity

Holding Cost
Why Order Cost Decreases

- Cost is spread over more units

Example: You need 1000 microwave ovens

<table>
<thead>
<tr>
<th>Description</th>
<th>Qty.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microwave</td>
<td>1000</td>
</tr>
</tbody>
</table>
EOQ Model

Annual Cost

Order Quantity

Order (Setup) Cost

Holding Cost
EOQ Model

Annual Cost

Total Cost Curve

Holding Cost

Order (Setup) Cost

Order Quantity
EOQ Model

Annual Cost

Total Cost Curve

Holding Cost

Order (Setup) Cost

Optimal Order Quantity (Q*)

Order Quantity
**EOQ Formula Derivation**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Annual demand (units)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Cost per unit ($)</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>Order quantity (units)</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Cost per order ($)</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Holding cost (%)</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Holding cost ($)</td>
<td>I x C</td>
</tr>
</tbody>
</table>

**Total cost**

\[
\text{Total cost} = \frac{Q}{2} \times I \times C + S \times \left(\frac{D}{Q}\right)
\]

**Number of Orders**

\[
\text{Number of Orders} = \frac{D}{Q}
\]

**Ordering costs**

\[
\text{Ordering costs} = S \times \left(\frac{D}{Q}\right)
\]

**Average inventory**

\[
\text{Average inventory units} = \frac{Q}{2}
\]

\[
\text{Average inventory $} = (\frac{Q}{2}) \times C
\]

**Cost to carry average inventory**

\[
\text{Cost to carry average inventory} = \frac{Q}{2} \times I \times C
\]

\[
= \frac{Q}{2} \times H
\]

**Take the 1st derivative**

\[
d(TC)/d(Q) = \frac{(I \times C)}{2} - \frac{(D \times S)}{Q^2}
\]

**To optimize**

\[
d(TC)/d(Q) = 0
\]

\[
\frac{DS}{Q^2} = \frac{IC}{2}
\]

\[
\frac{Q^2}{DS} = 2 / IC
\]

\[
Q^2 = (DS \times 2) / IC
\]

\[
Q = \sqrt{\frac{2DS}{IC}}
\]
Economic Order Quantity

\[ EOQ = \sqrt{\frac{2 \times D \times S}{H}} \]

D = Annual demand (units)
S = Cost per order ($)
C = Cost per unit ($)
I = Holding cost (%)
H = Holding cost ($) = I \times C
EOQ Model Equations

**Optimal Order Quantity**

\[ Q^* = \sqrt{\frac{2DS}{H}} \]

**Expected Number Orders**

\[ N = \frac{D}{Q^*} \]

**Expected Time Between Orders**

\[ T = \frac{\text{Working Days / Year}}{N} \]

\[ d = \frac{D}{\text{Working Days / Year}} \]

**ROP**

\[ \text{ROP} = d \cdot L \]

- **D** = Demand per year
- **S** = Setup (order) cost per order
- **H** = Holding (carrying) cost
- **d** = Demand per day
- **L** = Lead time in days
You’re a buyer for SaveMart.

SaveMart needs 1000 coffee makers per year. The cost of each coffee maker is $78. Ordering cost is $100 per order. Carrying cost is 40% of per unit cost. Lead time is 5 days. SaveMart is open 365 days/yr.

What is the optimal order quantity & ROP?
SaveMart EOQ

\[ EOQ = \sqrt{\frac{2 \times D \times S}{H}} \]

\[ \begin{align*} 
D &= 1000 \\
S &= $100 \\
C &= $ 78 \\
I &= 40\% \\
H &= C \times I \\
H &= $31.20 \\
\end{align*} \]

\[ EOQ = \sqrt{\frac{2 \times 1000 \times $100}{$31.20}} \]

\[ EOQ = 80 \text{ coffeemakers} \]
ROP = demand over lead time
    = daily demand x lead time (days)
    = \( d \times l \)

\[ D = \text{annual demand} = 1000 \]
\[ \text{Days / year} = 365 \]
\[ \text{Daily demand} = \frac{1000}{365} = 2.74 \]
\[ \text{Lead time} = 5 \text{ days} \]

\[
\text{ROP} = 2.74 \times 5 = 13.7 \Rightarrow 14
\]
Avg. CS = OQ / 2
= 80 / 2 = 40 coffeemakers
= 40 x $78 = $3,120

Inv. CC = $3,120 x 40% = $1,248

Note: unrelated to reorder point
Economic Order Quantity

\[ EOQ = \sqrt{\frac{2 \times D \times S}{H}} \]

- \( D \) = Annual demand (units)
- \( S \) = Cost per order ($)
- \( C \) = Cost per unit ($)
- \( I \) = Holding cost (%)
- \( H \) = Holding cost ($) = \( I \times C \)
What if …

1. Interest rates go up?
2. Order processing is automated?
3. Warehouse costs drop?
4. Competitive product is introduced?
5. Product is cost-reduced?
6. Lead time gets longer?
7. Minimum order quantity imposed?