(1989), Ho, Stapleton, and Subrahmanyam (1993a, 1993b), and others on the numerical valuation of options with multiple state variables.

In the special case where all of the volatilities are constant, the model is "path independent." This term-structure volatility function yields the constant volatility Ho and Lee (1986) model. In this case, we can obtain accurate option values for much longer maturities.

To illustrate our model, we study two applications: long-dated currency warrants and "cross-rate swaps." As benchmarks for this analysis, we estimate the necessary model parameters from historical exchange rate and interest rate data under the assumption that the exchange rate is lognormal and the volatility of forward interest rates is constant across time and maturity. We conduct sensitivity analysis with alternative assumptions about the form of the forward interest rate volatility functions. They are permitted to be in the constant elasticity of variance (CEV) class.

Using the benchmark parameter estimates, we find that the warrant values are quite sensitive to interest rate risk. For example, the large empirical biases in currency warrant premia demonstrated by Rogalski and Seward (1991) can be significantly reduced by incorporating interest rate risk. For cross-rate swaps whose payoffs are determined by foreign interest rates but paid in dollars at a 1:1 (fixed) exchange rate, the foreign interest rate process-related parameters are much more important than are the domestic interest rate parameters.

An outline of the article is as follows. We set up the general model in Section 1 and derive the valuation relationships in Section 2. In Section 3, we develop the general path-dependent model. This model is specialized with specific volatility functions to yield a path-independent model in Section 4. In Section 5, we provide details about the numerical implementation of the model developed in Sections 3 and 4. The data and empirical procedures used to estimate the model are introduced in Section 6. In Section 7, we analyze currency warrants and cross-rate swaps. Section 8 concludes.

1. Interest Rate and Foreign Currency Dynamics

We will use the risk-neutral valuation technique developed by Cox and Ross (1976) and Harrison and Pliska (1981). Let Q be the probability measure governing the evolution of all state variables in our economy. Assuming that our economic model is complete and arbitrage free, there exists a unique probability measure, called the *risk-neutral measure* and denoted as $\tilde{Q}$, such that the expected rate of return *in dollar terms* (units of the domestic currency) on every traded security equals the domestic (dollar) spot interest rate in each period.

Under $\tilde{Q}$, the dollar price of any contingent claim can be written as the expectation of its dollar payoffs discounted on a period by period basis by the domestic spot interest rate. The original probability measure, Q, is irrelevant for valuation. Therefore, to reduce extraneous detail, we study the model only under $\tilde{Q}$.

1.1 Term structure economies

We assume that trades occur after every h units of time, up to some finite horizon, τ (i.e., at times specified by the set: $[0, h, 2h, \ldots, \tau]$). At each trading date, we assume that investors can trade domestic and foreign discount bonds of all future maturities. At its maturity date, a domestic (foreign) discount bond pays one dollar (one unit of the foreign currency). In this section, we define the dynamic behavior of these bond prices by specifying the evolution of forward interest rates in the domestic and foreign economies.

Let $f_d(t, T)$ be the domestic forward interest rate contracted at time t for one period (of duration h) of borrowing or lending at time T. These rates are specified per unit of time. Further, the subscript d denotes that the variables are defined in the domestic economy. Corresponding terms in the foreign economy are denoted with a subscript f.

We first characterize the domestic term structure economy by specifying the evolution of forward interest rates using a discrete-time version of HJM.

Assumption 1 (Domestic Forward Rate Dynamics). *At each trading date, t, forward interest rates of all future maturities $T > t$ follow the stochastic difference equation:*

$$f_d(t+h, T) - f_d(t, T) = \alpha_d(t, T, \cdot)h = \sigma_d(t, T, \cdot)X_d(t+h)\sqrt{h}. \quad (1)$$

where $X_d(ih)$, $i = 1, 2, \ldots$ is a sequence of independent random variables with mean zero and variance one under $\tilde{Q}$. We follow the convention that the value of the random variable $X_d(t)$ is realized before any trading at date t but after any trading date $t - h$.

$\alpha_d(\cdot)$ and $\sigma_d(\cdot)$ are some functions that define the drift and variance of forward interest rates. They can also be functions of the current or past values of forward rates. Further, for market completeness, we assume that the number of states in the distribution of $X_d(t)$ is less than or equal to the number of distinct maturity bonds that trade at date $t - h$.

Specification (1) ensures consistency with the continuous-time model of HJM and also simplifies mean and variance calculations. The single period drift and variance of forward rates are given by $\alpha_d(\cdot)h$