International Investment Restrictions and Factor Pricing

James N. Bodurtha, Jr.
Georgetown University

August 1998

Last revised: July 1999

Abstract

Motivated by the original Black (1974), Adler-Dumas (1975), Subrahmanyam (1975) and Stulz (1981) restricted international capital asset pricing models (ICAPM), we extend Connor's (1984) well-diversified portfolio and factor pricing construct to the restricted international market case. Analogous to the Errunza-Losq (1985) partially restricted ICAPM, we identify the pricing implications of multi-factor risk structures in a partially restricted factor economy. To link the restricted market and factor pricing constructs, we admit 'local' or country-specific well-diversified factor portfolios. As in Connor's 'global' well-diversifed factor portfolio case, we find that globally-traded (internationally unrestricted) factor risks have equal prices across investors (countries). However, binding cross-country investment restrictions and locally well-diversified factor portfolios result in ***identical factor risks*** that have ***different factor prices*** across investors (countries). Therefore, well-diversified factor portfolios do not guarantee one set of global factor prices in restricted international financial markets.

For their comments, I thank Warren Bailey, Bob Korajczyk, Nelson Mark, Vijay Singal, Jay Shanken, Akhtar Siddique and René Stulz. This work has been supported by a Georgetown Univesity Summer Research Grant.

Correspondence – The McDonough School of Business, Georgetown University, Old North 313, 37th & O Streets, NW, Washington, DC 20057, phone: (202) 687-6351, fax: (202) 687-4031, e-mail: bodurthj@gunet.georgetown.edu

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Motivated by the original Black (1974), Adler-Dumas (1975), Subrahmanyam (1975) and Stulz (1981) restricted international capital asset pricing models (ICAPM), we extend Connor's (1984) well-diversified portfolio and factor pricing construct to the restricted international market case. Analogous to the Errunza-Losq (1985) partially restricted ICAPM, we identify the pricing implications of multi-factor risk structures in a partially restricted factor economy. To link the restricted market and factor pricing constructs, we admit 'local' or country-specific well-diversified factor portfolios. As in Connor's 'global' well-diversifed factor portfolio case, we find that globally-traded (internationally unrestricted) factor risks have equal prices across investors (countries). However, binding cross-country investment restrictions and locally well-diversified factor portfolios result in ***identical factor risks*** that have ***different factor prices*** across investors (countries). Therefore, well-diversified factor portfolios do not guarantee one set of global factor prices in restricted international financial markets.

Our model development follows extant work on international asset pricing under restrictions. This work has, almost uniformly, been set in the capital (reference) asset pricing model (CAPM) context. In this context, a reference pricing portfolio is identified and other investments are priced by their return covariance with the reference portfolio's return.

Table 1 summarizes 25 years of research on the international investment market integration-segmentation question. Among these works, Errunza-Losq (1985) is closest in focus to our work. Their work both models and tests the partial segmentation specification as a general case, which embodies integration and total segmentation at its extremes. Effectively, we generalize their CAPM-based approach within the unified beta pricing construct of Connor (1984).

In the "Findings" column of Table 1, we have underlined the study findings that have rejected the integration hypothesis. Among these studies, CAPM and factor or arbitrage pricing model tests have been conducted. More than half of the tabulated studies reject integration, while the other studies either maintain integration or support some mix of the integration and segmentation alternatives. These results clearly show the need for a specification that encompasses both integration and segmentation across single reference asset-based and multiple factor-based pricing models.

We divide our treatment of international financial market partial segmentation into five sections and an appendix. The first section develops an international partially-segmented markets asset pricing model. This approach, which is based on normal return and/or quadratic utility assumptions, has been used previously in the international capital asset pricing model literature.

In the second section, we augment this analysis, and consider implications of both ‘well-diversified’ investors [Connor (1984)] and factor portfolio trading restrictions. The third section contains an example of the modeled phenomena. The fourth section extends Connor’s integrated market factor pricing model framework to the partially restricted market case. Section five concludes the work, and our appendix specifies the example decision problem and its solution method.

1. An International Capital Asset Pricing Model

Among many potential stratifications, we segment the world into a domestic country and the rest of the world. We call the rest of the world the ‘foreign country.’ Across the domestic and foreign countries, we differentiate both investors and investments.

Investors domiciled in one country face restrictions on the investments that they may make in the other country. These restrictions differentiate investments by their ownership clienteles: domestic, D, and foreign, F.

Across domestic and foreign investor clienteles, four investment subsets are identified: domestic, domestic-international, foreign-international and foreign. These investment sets are identified by the subscripts D, I_D, I_F and F, respectively.

The allocation vector for investors in a class i is characterized as follows:

$$q^i = \left\{ q_D'^i \; q_{I_D}'^i \; q_{I_F}^i \; q_F'^i \right\}' \quad \text{for } i = D,F.$$

International market restrictions can be characterized by constraints on these allocation vectors. We have predominant interest in three cases: segmented markets, partially-segmented markets and integrated market.

For segmented markets, cross-country investments do not exist:

$$q^D = \left\{ q_D^{D\prime}\ q_{I_D}^{D\prime}\ 0'\ 0' \right\}, \quad q^F = \left\{ 0'\ 0'\ q_{I_F}^{F\prime}\ q_F^{F\prime} \right\}$$

For partially-segmented markets, domestic and foreign investors may buy and sell both sets of international investments:

$$q^D = \left\{ q_D^{D\prime}\ q_{I_D}^{D\prime}\ q_{I_F}^{D\prime}\ 0' \right\}', \quad q^F = \left\{ 0'\ q_{I_D}^{F\prime}\ q_{I_F}^{F\prime}\ q_F^{F\prime} \right\}'$$

For the integrated market, all allocation vectors are unrestricted:

$$q^D = \left\{ q_D^{D\prime}\ q_{I_D}^{D\prime}\ q_{I_F}^{D\prime}\ q_F^{D\prime} \right\}', \quad q^F = \left\{ q_D^{F\prime}\ q_{I_D}^{F\prime}\ q_{I_F}^{F\prime}\ q_F^{F\prime} \right\}'$$

With regard to the potential costs of investment restrictions, the predominant pricing implication is identified with marginal investors for each investment set. If foreign investors can buy domestic-international investments without restrictions, then the associated investment demands ($q_{I_D}^F$) satisfy the foreign investors' marginal conditions. In this eventuality, a reference pricing portfolio for domestic-international investments will include the foreign investor's holdings. Furthermore, the impact of foreign investmend demands for domestic-international investments will also impact purely domestically-held investments. Hence, the return structure of domestic investments is both directly and indirectly related to foreign investment returns.

If foreign investors are restricted in buying and selling domestic-international investments, then only domestic investor demands enter the domestic investment reference pricing portfolio. In this case, potential restrictions on domestic investors' purchases and sales of foreign-international investments are relevant. If unrestricted, then the domestic investment reference pricing portfolio must include domestic investors' holdings of foreign-international investments ($q_{I_F}^{D}$).

The differences between the integrated, partially-segmented, and segmented market cases are determined both by the relative capacity of investors to buy the international investments (indicated by $q_{I_D}^{F}$ and $q_{I_F}^{D}$) and by the underlying stochastic factor structure of returns. Particularly, globally-traded factor portfolios may substitute for restricted security trading. Together, both security specific restrictions on investors and factor structure differentiate the partially-segmented market from its two extrems, complete segmentation and full integration.

Under our investor attributes and market structure definitions, we derive an international CAPM in four sub-sections: decision problem statement, aggregation, partial segmentation case, and factor structure implications.

1.1 The Investor's Decision Problem

For the time t to time T period, representative foreign (i=F) and domestic (i=D) investors solve the following consumption-investment problem:

$$\max_{\{q^i(t)\}} \; U^i\left[w^i(t) - q^i(t)'p(t)\right] + \sum_{S=1}^{S} \pi(s,t)\, U^i\left[q^i(t)'x(s,T)\right] \qquad 1)$$

$$\text{s.t. } |q_R^i(t)| \le K_R^i(t)$$

$q^i(t)$ — the beginning of period investor i allocation vector, which is partitioned into unrestricted and restricted investments as $\left\{q_U^i(t),\, q_R^i(t)\right\}$, respectively.

U^i — the investor i utility function, $U^{i'}>0$, $U^{i''}<0$.

$w^i(t)$ — units of wealth owned by investor i at time t, which may be allocated to consumption, $c^i(t)$, or to investment in a subset of the n investments, $q^i(t)$, $w^i(t) = c^i(t) + q^i(t)'p(t)$.

$w^i(s,T)$ — units of wealth owned by investor i at time T in state s, which will be allocated to future consumption, $[c^i(s,T), = q^i(t)'x(s,T)]$. To simplify notation, $E[W^i(T)] = \sum_{s=1}^{S} \pi(s,t)\, U^i\,[q^i(t)'x(s,T)]$.

$p(t)$ — the price vector at time t for purchasing one unit of each of the n investments.

$c^i(t)$ — initial period t consumption of investor i, equaling $w^i(t) - q^i(t)'p(t)$.

$x(s,T)$ — the cashflow vector received at time T in state s (of S total states) from owning one unit of each of the n investments:

$$= \alpha(t) + e(s,T) \qquad 2)$$

$$= \alpha(t) + \beta f(s,T) + \varepsilon(s,T) \qquad 3)$$

To simplify notation, $E[x(T)] = \sum_{s=1}^{S} \pi(s,t)\, x(s,T)$

$\alpha(t)$ — the expected cashflow vector for the n investments.

$e(s,T)$ — the state-specific (composite) cashflow error. To simplify notation, $E[e(T)] = \sum_{s=1}^{S} \pi(s,t)\, e(s,T)$

β — the n x K dimensioned factor-weighting matrix, which is partioned into domestic and world factor components, $[\beta^d, \beta^w]$.

$f(s,T)$ — the K-dimension zero-expected value and time T factor innovation vector in state s. The factor vector is partioned into domestic $[f^d(s,T), 1 \times K^d]$ and world $[f^w(s,T), 1 \times K^w]$ factor subsets $(K = K^d + K^w)$. The general covariance matrix is $\Delta = \begin{bmatrix} \Delta_{dd} & \Delta_{dw} \\ \Delta_{wd} & \Delta_{ww} \end{bmatrix}$. To simplify our discussion, we assume that the world factors incorporate all common factor components with the domestic factors, and an orthogonal domestic and world factor covariance matrix results: $\Delta = \begin{bmatrix} \Delta_{dd} & 0 \\ 0 & \Delta_{ww} \end{bmatrix}$

$\varepsilon(s,T)$ — the idiosynactic component of the composite cashflow error. To simplify notation, $E[\varepsilon(T)] = \sum_{s=1}^{S} \pi(s,t)\, \varepsilon(s,T)$

$x_j(s,T)$ — an n_j subset of the investment cashflow vector, $j = D, I_D, I_F, F$.

$x(s,T) = [x_D(s,T)', x_{I_D}(s,T)', x_{I_F}(s,T)', x_F(s,T)']'$.

$x_j(s,T) = \alpha_j(t) + e_j(s,T)$

$x_j(s,T) = \alpha_j(t) + \beta_j f(s,T) + \varepsilon_j(s,T)$

$\alpha_j(t)$ — an n_j subset of the investment expected cashflow vector, $j = D, I_D, I_F, F$.

$\alpha(t) = [\alpha_D(t)', \alpha_{I_D}(t)', \alpha_{I_F}(t)', \alpha_F(t)']'$.

$e_j(s,T)$ — an n_j subset of the investment cashflow error vector, $j = D, I_D, I_F, F$.

$e(s,T) = [e_D(s,T)', e_{I_D}(s,T)', e_{I_F}(s,T)', e_F(s,T)']'$.

β_j	a n_j subset of the beta factor weighting matrix, j = D, I_D, I_F, F. $\beta(t) = [\beta_D(t)', \beta_{I_D}(t)', \beta_{I_F}(t)', \beta_F(t)']'$.
$\varepsilon_j(s,T)$	an n_j subset of the the idiosynactic component of the investment cashflow error vector, j = D, I_D, I_F, F. $\varepsilon(s,T) = [\varepsilon_D(s,T)', \varepsilon_{I_D}(s,T)', \varepsilon_{I_F}(s,T)', \varepsilon_F(s,T)']'$.
$p_j(t)$	an n_j subset of the investment price vector, j = D, I_D, I_F, F. $p(t) = [p_D(t)', p_{I_D}(t)', p_{I_F}(t)', p_F(t)']'$.
$q_j(t)$	the aggregate beginning of period allocation vector for the n_j investments in investment subset j = D, I_D, I_F, F: $q(t) = [q_D(t)', q_{I_D}(t)', q_{I_F}(t), q_F(t)']'$.
$K^i_R(t)$	the upper and lower bound (vector) constraint on the restricted investment set.
m^i_j	the investor subset i owning (unrestricted for) investment j, i=D or F and j= D, I_D, I_F, F
μ^i_j	the number of investors in set m^i_j

The first order conditions for the unrestricted investments are

$$-U^{i'}(c^i(t))p(t) + \sum_{s=1}^{S} \pi(s,t)\, U^{i'}[q^i(t)'x(s,T)]\, x(s,T) = 0 \qquad 4)$$

And for restricted investments,

$$-U^{i'}(c^i(t))p(t) + \sum_{s=1}^{S} \pi(s,t)\, U^{i'}[q^i(t)'x(s,T)]\, x(s,T) + T^i_R = 0 \qquad 5)$$

Where T^i_R is the n_R vector of constraint multipliers, with n_R being the number of restricted investments (n_R<n).

Substituting for the expectation or the product of marginal utility and the investment cashflow, equation 4) becomes

$$-U^{i'}(c_i(t))p(t) + \mathrm{cov}\big[U^{i'}[q^i(t)'x(T)]x(T)\big] + E\big[U^{i'}[q^i(t)'x(T)]\big]\,\alpha(t) = 0 \qquad 6)$$

1.2 Aggregation

In order to derive the aggregate marginal condition that applies to the investment return vector, we may make one of two assumptions: investors have quadratic utility and/or returns are distributed jointly normal. Both of these assumptions imply that only the first two moments of the joint returns distribution are relevant for investor decisions.

Under normality, aggregation follows Rubinstein (1973). In this case,

$$\mathrm{cov}\big[U^{i'}[q^i(t)'x(T)]x(T)\big] = -E\big[U^{i''}[q^i(t)'x(T)]\big]\mathrm{cov}\big[q^i(T)'x(T),x(T)\big] \qquad 7)$$

Under quadratic utility, the aggregation conditions follow Rubinstein (1974):

$$U\big[w^i(s,T)\big] = b^i q^i(t)'x(s,T) - c\big[q^i(t)'x(s,T)\big]^2 \qquad 8)$$

The c coefficients, which are associated with the squared wealth term and risk aversion, are assumed to be equal across investors.

We posit both of these cases to highlight the continuous-state returns distribution and discrete-state returns distribution cases that yield our final specification. These two cases are analogous to the limit and finite economy cases analyzed by Connor (1984). With investment restrictions, this less general framework is useful to construct an example of the type historically evaluated in asset pricing tests. We use this specification to link the restricted factor pricing model with the restricted investment capital asset pricing model. We treat the general case in section 4.

To proceed, we assume that buying and selling a riskless investment, identified by the subscript o, is possible. Therefore, $e_o(s,T) = 0$ for all s in S.[1] We have

$$U^{i'}[c_i(t)]\,P_o(t) = E\big[U^{i'}[q^i(t)'x(T)]\big] \qquad 9)$$

Substituting this quantity into the unrestricted first order condition, 6), we apply our normal returns distribution or (alternatively) quadratic utility assumption. (In the covariance definition, the drift components $[\alpha(t)]$ of the return vector $[r(T)]$ are constants, and drop out in both cases.)

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[1] Chamberlain and Rothschild (1983) treat the single economy case in which the riskless investment is available only in the large limit economy.

Under the normal returns distribution assumption,

$$E[U^{i'}(w^i(T))]\,[\alpha(t)- \frac{p(t)}{p_o(t)}] - E[U^{i''}(w^i(T))]\mathrm{cov}[q^i(t)'e(T),e(T)] = 0 \qquad 10)$$

For the quadratic investor utility case, a more particular specification results:

$$E[U^{i'}(w^i(T))] = b^i - 2cE\,[w^i(T)], \text{ and } E[U^{i''}(w^i(T))] = 2c$$

On rearranging equation 10),

$$\lambda^i\,[\alpha(t) - \frac{p(t)}{p_o(t)}] = \mathrm{cov}\,[q^i(t)'e(T),e(T)], \qquad 11)$$

$$\lambda^i = E\,[U^{i'}(w^i(T))] \,/\, E[U^{i''}(w^i(T))]$$

In order to derive the aggregate pricing relation, we recognize that unrestricted investors set prices. Therefore, we aggregate each element of the first order condition vector over the set of domestic (i=D) and/or foreign (i=F) investors unrestricted (at the margin), m^i_j, in holding the respective investment, j. Note that the set of unrestricted investors differs across investments[2].

We aggregate over the first order conditions of investors holding an investment j. Three investment-type associated aggregates are of interest for j={D, I_D, I_F and F}: prices of risk, reference wealth portfolios and investment allocation vectors.

$$\lambda_j = \sum_{i\in\{m^D_j, m^F_j\}} \lambda^i$$

$$w_j(s,T) = q_j(t)'r(s,T)$$

$$q_j(t) = \sum_{i\in\{m^D_j, m^F_j\}} q^i(t)$$

2 Positive definiteness of each representative investors' unrestricted investment cashflow covariance matrix provides sufficient second order conditions.

Therefore, each investment class price of risk, λ_j, and reference wealth portfolio, $w_j(T)$, may differ.

$$\lambda j \left[\alpha_j(t) - \frac{p_j(t)}{p_o(t)}\right] = \text{cov}\left[q_j(t)'e(T), e_j(T)\right] \text{ for all j.} \qquad 12)$$

From both a theoretical and empirical perspective, pricing relation 12) is vacuous. Each investment is priced idiosyncratically. If we are to add content to this model, then some cross-investment price of risk and reference pricing portfolio weight restrictions are necessary. Our hypothesized investor attributes and market structure provide these restrictions.

1.3 Partially-segmented Market Implications

The partially-segmented model includes both the full integration and complete segmentation models as its extreme sub-cases. Hence, we focus our model development on partially-segmented markets.

Given our definitions of investor attributes and market structure, we identify four investment pricing relations. These equations are restrictions on the fully idiosyncratic pricing relation, 12). The restrictions fall on the prices of risk, λ_j, and the reference pricing portfolio weights, $q_j(t)$.

The four restrictions associated with each investment set are the following:

Investment Set	Price of Risk (λ_j)	Reference Portfolio Weight ($q_j(t)$)
Domestic	$\lambda_D = \sum_{i \in \{m_D^D\}} \lambda^i$	$q_D(t) = \sum_{i \in \{m_D^D\}} q^i(t)$
Domestic-International	$\lambda_{I_D} = \sum_{i \in \{m_{I_D}^D, m_{I_D}^F\}} \lambda^i$	$q_{I_D}(t) = \sum_{i \in \{m_{I_D}^D, m_{I_D}^F\}} q^i(t)$
Foreign-International	$\lambda_{I_F} = \sum_{i \in \{m_{I_F}^D, m_{I_F}^F\}} \lambda^i$	$q_{I_F}(t) = \sum_{i \in \{m_{I_F}^D, m_{I_F}^F\}} q^i(t)$
Foreign	$\lambda_F = \sum_{i \in \{m_F^F\}} \lambda^i$	$q_F(t) = \sum_{i \in \{m_F^F\}} q^i(t)$

Substituting from these definitions into each cross-section relation, 12), and stratifying by investment type:

$$\lambda_j\left[\alpha_j(t)-p_j(t)/p_o(t)\right]=\mathrm{cov}\left[q_j(t)' e(t), e_j(t)\right] \text{ for } j=D, I_D, I_F, F \qquad 13)$$

With the investors' reference portfolios being traded, we stack these four equation systems. To aggregate these pricing relations, we identify two pricing relations for the two representative investor clienteles: domestic and foreign. For the domestic investor allocation, $q^D(t)$, marginal condition 13) must be satisfied for domestic investors' allocations (j= {D, I_D, I_f}).

$$\lambda^D\left[\alpha_j(t)-p_j(t)/p_o(t)\right] = \mathrm{cov}\left[q^D(t)' e(t), e_j(t)\right] \text{ for } j=D, I_D, I_F \qquad 13)$$

Pre-multiplication of both sides of this vector equation by the transpose of the domestic allocation vector solves for the respective price of risk:

$$\lambda^D = \frac{var[q^D(t)'e(T)]}{q^D(t)'\alpha(t) - q^D(t)'p(t)/p_o(t)} \tag{14)}$$

We substitute this quantity into equation 13):

$$\alpha_j(t) - \frac{p_j(t)}{p_o(t)} = \left[\frac{q^D(t)'\alpha(t) - q^D(t)'p(t)/p_o(t)}{var[q^D(t)'e(T)]}\right] cov\,[q^D(t)'e(T), e_j(T)] \quad \text{for } j \in \{D, I_D, I_F\} \tag{15-a)}$$

Analogously, we define the foreign investors' marginal conditions:

$$\alpha_j(t) - \frac{p_j(t)}{p_o(t)} = \left[\frac{q^F(t)'\alpha(t) - q^F(t)'p(t)/p_o(t)}{var[q^F(t)'e(T)]}\right] cov\,[q^F(t)'e(T), e_j(T)] \quad \text{for } j \in \{I_D, I_F, F\} \tag{15-b)}$$

Redefining these cashflow-pricing relations as expected return relations,

$$E\left[r_j(t)\right] - r_o(t) = \left(E\left[r_p^D(t)\right] - r_o(t)\right)B_j^D \qquad j \in \{D, I_D, I_F\} \tag{16-a)}$$

$$E\left[r_j(t)\right] - r_o(t) = \left(E\left[r_p^F(t)\right] - r_o(t)\right)B_j^F \qquad j \in \{I_D, I_F, F\} \tag{16-b)}$$

$$r_{jk}(T) = \frac{\alpha_{jk}(t) + \beta_{jk} f(s,T) + \varepsilon_{jk}(s,T)}{p_{jk}(t)} - 1 \qquad \text{for } j \in \{D, I_D, I_F, F\},\ k = \{1, \ldots, n_j\}$$

$$r_j(T) = \left\{r_{1_j}(T), \ldots, r_{n_j}(T)\right\}', \qquad j \in \{D, I_D, I_F, F\}$$

$$r(T) = \left\{r_D(T)', r_{I_D}(T)', r_{I_F}(T)', r_F(T)'\right\}'$$

$$r_o(t) = \frac{1}{p_o(t)} - 1$$

$$r_p^i(T) = q^i(t)'\,r(T) \qquad i = D, F$$

$$\alpha_p^i(t) = q^i(t)'\,\alpha(t) \qquad i = D, F$$

$$e_p^i(T) = q^i(t)'\,e(T) \qquad i = D, F$$

$$B_{jk}^{i} = \frac{\operatorname{cov}\left[e_p^i(T), e_{jk}(T)\right]}{\operatorname{var}\left[e_p^i(T)\right]} \qquad i=D,\ j\in\{D,I_D,I_F\},\ k=\{1,\ldots,n_j\}$$

$$i=F,\ j\in\{I_D,I_F,F\},\ k=\{1,\ldots,n_j\}$$

$$B_j^i = \left\{B_{1_j}^{i\,\prime},\ldots,B_{n_j}^{i\,\prime}\right\}^{\prime} \qquad i=D,\ j\in\{D,I_D,I_F\}$$

$$i=F,\ j\in\{I_D,I_F,F\}$$

In equation 16) one variant of the partially restricted international CAPM is stated. In this form, the restrictions imply that different 'local market' reference-pricing portfolios are necessary. These reference-pricing portfolios are the aggregate wealth portfolios in a local market. The local reference pricing portfolio returns are $r_p^D(T)$ and $r_p^F(T)$ for the domestic and foreign markets, respectively.

Under the partially-segmented CAPM, a set of alternative pricing hypotheses result. These hypotheses are differentiated by investor attribute and market structure definitions. Under complete segmentation, only domestic investments enter the domestic reference pricing portfolio, and only foreign investments enter the foreign reference pricing portfolio.

Under partial segmentation, investors adjust their investments to account for their access to international investments. Hence, unrestricted international investor allocations ($q_{I_F}^D$ for the domestic market and $q_{I_D}^F$ for the foreign market) add to the reference-pricing portfolios. As a result, two pricing relations must be satisfied for the internationally traded investments (I_D,I_F). Under integration, all investments enter the reference pricing portfolio, which is the world market portfolio, and all investments are priced by a single global CAPM.

1.4 Factor Structure Implications

Our market factor structure and factor risk diversification implications follow the work of

Connor (1984).[3] To develop linear pricing models that admit market restrictions, we proceed under our two previous sets of conditions: normally-distributed returns and/or quadratic utility.

For the two investor clientele pricing relations, 15), a factor-based cashflow definition, 3), implies the following:

$$e_p^i(s,T) = \beta_p^i f(s,T) + \varepsilon_p^i(s,T) \qquad \beta_p^i = q^i(t)'\beta,\ \varepsilon_p^i(T) = q^i(t)'\varepsilon(T) \quad \text{for } i=\{D,F\} \qquad 17)$$

When the reference pricing portfolios are not well-diversified, local market CAPM analysis necessitates identification of the reference pricing portfolio. The Connor factor pricing approach cannot be applied.

For the CAPM, less than full reference-portfolio diversification implies the following:

$$\mathrm{Var}\,[e_p^i(T)] = \beta_p^i \,\Delta\, \beta_p^{i\prime} + \sigma_{\varepsilon_p}^{i^2} \qquad \text{for } i=\{D,F\} \qquad 18)$$

$$\sigma_{\varepsilon_p}^{i^2} = \mathrm{var}\,[\varepsilon_p^i(T)]$$

$$\mathrm{Cov}\,[e_p^i(T), e(T)] = \beta_p^i \,\Delta\, \beta_j + \beta_{\varepsilon\varepsilon_j^p}\, \sigma_{\varepsilon_p}^{i^2} \qquad \text{for } i=\{D,F\}$$

$$\beta_{\varepsilon\varepsilon_j^p} = \mathrm{cov}\,[\varepsilon_p^i(T),\ \varepsilon(T)] \,/\, \sigma_{\varepsilon_p}^{i^2}$$

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3 In section 4, we develop a Connor-like restricted markets factor pricing model. The pricing implications depend upon the factor structure of restricted and unrestricted investment returns and investor access to the associated investments. These implications are also present in our hybrid model. Furthermore, a fully factor-based model will be indistinguishable empirically from certain specifications of the factor-augmented CAPM that we develop. The multi-factor approach begins with Merton (1973) and Ross (1974, 1976). Subsequent developments include Brock (1982), Huberman (1982), Chamberlain (1983), Chamberlain and Rothschild (1983), Dybvig (1983), Grinblatt and Titman (1983) , Bossaerts and Green (1989), Connor and Korajczyk (1989) and Bansal and Viswanathan (1993). Shanken (1987) provides an alternative approach..

On substitution into the CAPM equation, 15)

$$\alpha_j(t) - \frac{p_j(t)}{p_o(t)} = \beta_j^* \gamma_D^*, \qquad j \in \{D, I_D, I_F\} \qquad \text{19-a)}$$

$$\alpha_j(t) - \frac{p_j(t)}{p_o(t)} = \beta_j^* \gamma_F^*, \qquad j \in \{I_D, I_F, F\} \qquad \text{19-b)}$$

$$\beta_j^* = [\beta_j : \beta_{\varepsilon\varepsilon_j^p}] \text{ and } \varepsilon_p^i(s,T)\gamma_i^* = \begin{bmatrix} \gamma_{d_i}^* \\ \gamma_{w_i}^* \\ \gamma_{\varepsilon_p}^i \end{bmatrix} = \begin{bmatrix} \dfrac{\alpha_p^i(t) - p_p^i(t)/p_o(t)}{\beta_p^i \, \Delta \, \beta_p^{i'} + \sigma_{\varepsilon_p}^{i^2}} \end{bmatrix} \begin{bmatrix} \Delta \, \beta_p^{i'} \\ \sigma_{\varepsilon_p}^{i^2} \end{bmatrix}$$

The gamma $^{(*)}$ parameters are functions of aggregate quantities alone. For unconditional CAPM tests, these parameters can be treated as constants. However, the reference portfolio must be identified in order to calculate the $\beta_{\varepsilon\varepsilon_j^p}$ coefficient. If this portfolio is known, then this ‘factor’ model and the corresponding CAPM, 15), are equivalent. Of course, idenfication of the reference pricing portfolio implies identification of this reference portfolio residual risk.

If the reference-pricing portfolio residual risk is assumed to be 'well diversified' in the Connor-sense, then pricing relationship 19) simplifies. A reference pricing-portfolio is well-diversified when it is spanned exactly by the factors.

2. 'Well-Diversified' Reference-Pricing Portfolios

If reference-pricing portfolios are well-diversified, then the undiversified risk, $\varepsilon_p^i(s,T)$, in equation 17), equals zero.[4] The scalar variance of the reference portfolio return is the factor loading beta matrix-weighted factor covariance matrix, Δ.

$$\text{Var}\,[e_p^i(T)] = \beta_p^i \, \Delta \, \beta_p^{i'} \qquad \text{for } i=\{D,F\} \qquad 20)$$

4 Wei (1988) suggests augmenting the factor set with the market portfolio as a means to induce the well-diversified investors and exact factor pricing.

The covariance vector is the following:

$$\mathrm{Cov}\,[e^{i}_{p}(T), e(T)] \;=\; \beta^{i}_{p}\,\Delta\,\beta_{j} \qquad \text{for } i=\{D,F\} \qquad 21)$$

Under these assumptions, the presence of a factor structure for returns necessitates consideration of international investor activity to arbitrage across markets. Since we have assumed that the reference pricing portfolios are well-diversified, Connor's factor pricing model results apply. Investors' positions will be constructed from linear combinations of 'locally well-diversified' factor mutual funds and the riskless investment. Therefore, international investors' ability to trade any factor mutual fund across markets (either directly or by portfolio formation) implies that the associated global factor prices of risk must be equal across markets.

To better motivate our discussion on this point, we define two orthogonalized factor sets: domestic investment-related, f^{d_i}, and all investment-related, f^{w}; for both domestic (i=D) and foreign (i=F) investors. The associated risk prices are γ^{d_i} and γ^{w}, respectively.

The factor pricing model is the following:

$$\begin{bmatrix} \alpha_i(t) \\ \alpha_{I_D}(t) \\ \alpha_{I_F}(t) \end{bmatrix} - \begin{bmatrix} p_i(t)/p_o(t) \\ p_{I_D}(t)/p_o(t) \\ p_{I_F}(t)/p_o(t) \end{bmatrix} = \begin{bmatrix} \beta^{d_i} & \beta^{w_i} \\ 0 & \beta^{w}_{I_D} \\ 0 & \beta^{w}_{I_F} \end{bmatrix} \begin{bmatrix} \gamma^{d_i} \\ \gamma^{w} \end{bmatrix} \qquad \text{for } i=\{D,F\} \qquad 22)$$

$$\gamma^{i} = \begin{bmatrix} \gamma^{d_i} \\ \gamma^{w} \end{bmatrix} = \left[\frac{\alpha^{i}_{p}(t) - p^{i}_{p}(t)/p_o(t)}{\beta^{i}_{p}\,\Delta\,\beta^{i'}_{p}} \right] \Delta\,\beta^{i'}_{p} \qquad 23)$$

Note that the gammas (factor risk prices) are functions only of aggregate quantities. These factor risk prices are differentiated in a local market by the reference pricing portfolio factor betas. Such a factor pricing relation requires identification of the <u>exact</u> factor set. If we are unable to identify the reference-pricing portfolio, then this linear security pricing relation holds when the reference portfolio return is spanned by the factors.

With well-diversified allocations, $\mathrm{Var}\,[e_p^i(T)] = \beta_p^i \, \Delta \, \beta_p^{i\prime}$, equations 23) and 14) together imply the following factor risk prices:[5]

$$\gamma^i = \Delta\, \beta_p^{i\prime} \big/ \lambda^i \qquad \text{for } i = \{D, F\} \qquad 24)$$

[5] Given risk aversion ($\lambda^i > 0$) and nonsingular factor risks (Δ is positive definite), a factor price (γ^i) is nonzero if and only if the investment's marginal investor cannot fully diversify the factor risk ($\beta_p^{i\prime} \neq 0$).

To increase the power associated with testing factor model specification 22), we can follow Shanken and Weinstein (1990). They suggest incorporating the pricing relation on any traded factor portfolios. For these factor portfolios, the associated β coefficients are one for the associated factor risk premium and zero for the other factor risk premia.

We write $\alpha_{f_i}(t)$ as the drift vector for the traded factor portfolios (available to investors in class i). We also partition the factor risk premia into traded and non-traded subsets, which are indicated by T and N subscripts, respectively, $\gamma = \{\gamma_T, \gamma_N\}$. If we similarly order the beta matrix, $\beta_j = [\beta_{T_j}\ \beta_{N_j}]$, then

$$\begin{bmatrix} \alpha_i(t) \\ \alpha_{I_D}(t) \\ \alpha_{I_F}(t) \end{bmatrix} - \begin{bmatrix} p_i(t)/p_o(t) \\ p_{I_D}(t)/p_o(t) \\ p_{I_F}(t)/p_o(t) \end{bmatrix} = \begin{bmatrix} \beta_{d_i} \\ \beta_{I_D} \\ \beta_{I_F} \end{bmatrix} \begin{bmatrix} \alpha_{f_i}(t) - p_{f_i}(t)/p_o(t) \\ \gamma_{N_i} \end{bmatrix} \qquad \text{for } i=\{D,F\} \qquad 25)$$

Pricing equations 22) [and 25)] remain local market pricing relations. Without further specification, we cannot identify the cross-market factor price relationships. Clearly, and as specified in equation 22), any world market factor must be priced equally across markets. As a result, global market clearing must equate the globally priced factors, equation 23). No such restriction applies to the purely domestically traded factor prices, and their prices will, generally, differ. In the next section, we construct an example to highlight this fact. In the subsequent section, we prove the general case in which our example supplies a needed contradiction.

3. An Example

To focus and illustrate the results of the previous section, we have constructed an example. The example is outlined in Tables 2-4. At the top of Table 2, the cashflow generating process is defined (as in equation 3). There are four cashflow components: expected cashflow, factor betas, factor innovations and idiosyncratic cashflow innovations. A discrete state-space is defined, and this state-space satisfies the conditions defined in Connor (1984) for a well-diversified finite economy equilibrium. Particularly, the cashflow innovations are mean zero, diversifiable and independent of the factor innovations.

Among investments, the first foreign (F_1) and domestic (D_1) investments are relatively high-risk. This risk comes from two sources: the high variability of investment specific innovations and their relatively high betas. To differentiate the investor opportunity sets, the riskiest foreign investment has a higher expected cashflow (10) than the riskiest domestic investment (4). The two international investments (I_1 and I_2) have both moderate risk and expected cashflows.

The remaining domestic and foreign investments (D_2 and F_2) serve as diversifying investments. Their investment-specific cashflow innovations are constructed to offset the idiosyncratic innovations in the other investments. This investment cashflow innovation construction is outlined in the appendix.

The second domestic investment (D_2) diversifies domestic investors who hold the first domestic investment and the two international investments. Analogously, the second foreign investment (F_2) diversifies foreign investors who hold the first foreign investment and the two international investments. Both diversifying investments have moderate idiosyncratic risk and no factor risk.

Table 3 presents the total cashflow covariance matrix for six investments and two factors. Additionally, we note that our orthogonal factor innovation and investment cashflows structure will have no implications for our main results.

Table 4 characterizes our two investor sets, the associated investment opportunities and the market-clearing results. To ensure that both CAPM and factor economy equilibria are feasible, we assume quadratic utility as suggested by Dybvig and Ingersoll (1982). The specific utility functions are defined at the top of Table in Panel A:. To derive market-clearing, we create a pareto-optimizing super-investor who solves an equally-weighted utility function for representative investors in the two countries.[6] Coming to market, the domestic and foreign investors have equal holdings of the traded investments and they each own all of their domestic market-specific investment. These initial allocations are indicated in Table 4: Panels B and C.

We consider two market clearings. In the first market, the diversifying investments (D_2 and F_2) are not traded. As a result, the associated economy is not well-diversified. The optimal investment allocations are listed in Table 4-Panel B, and the associated investment and factor-mimicing investment

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6 This approach has also been implemented by Dumas (1989).

prices are listed in Panel D. Among these investments, the riskless investment and the international investments are unrestricted investments and are traded by both domestic and foreign investors. Hence, the prices of these commonly traded investments are equivalent for both investors.

In this restricted markets case, investments that do not directly trade or indirectly trade through factor portfolio substitution in both countries have different prices. Because restricted investment prices are set by the investors who may own them, the foreign investment (F_1) and domestic investment (D_1) prices are set by foreign and domestic investors, respectively. The two different prices that are listed for the restricted investments by domestic and foreign investors are the unrestricted price plus the investment restriction shadow price.

Of greatest importance are the costs of the factor portfolios. We see that the costs of the domestic and world factor portfolios differ across countries. (Note that the factor costs are negative because the factor innovations have zero expected value and are positively correlated with end of period wealth.) Analogously, Tale 4 - Panel E reports the undiversified market risk prices. Given two different investment opportunity sets, the market prices of risk differ across countries. Consistent with the differeing factor costs, the market clearing factor prices of risk differ also.

The well-diversified economy case is also highlighted in Table 4: Panels C, D and E. The Panel C and D results for the well-diversified economy are analogous to those already discussed for the undiversified economy. In our restricted investment context, the most important results are found in the last two columns of Panel D.

With a well-diversified market structure, world factor-mimicing portfolio costs are equivalent in both countries (-0.05). Nevertheless, and even though the domestic factor is the same in both domestic and foreign countries, the domestic factor-mimicing portfolio costs differ across countries (-0.075 and -0.08, respectively.)

Clearly, (locally) well-diversified market factor prices may differ across restricted international (or interregional) security markets. Finally, Table 4-Panel E also summarizes three salient characteristics of our locally well-diversified economy: heterogeneity of local CAPM prices of risk across countries ($\lambda^d \neq \lambda^f$), homogeneity of unrestricted factor risk price (γ^w) across countries and heterogeneity of restricted factor risk prices $\left(\gamma^{d_D} \neq \gamma^{d_F}\right)$ across countries.

4. The General Case

With our multivariate normal distribution or quadratic utility assumption example, we have shown how Connor's (1984) Unified Beta Pricing Theory treatment of Ross' original mutual fund separation and arbitrage pricing theory can be extended to the partially-segmented (international) markets case. Connor's work treats both finite factor economy and limit economy cases. As our example has a finite investment set, it is a finite factor economy special case. (Our example also has a finite state space).

Connor's results are based on the availability of a riskless asset, absence of arbitrage and weak utility restrictions (U'>0 and U"<0.) Importantly, his results apply directly to the case of 'well-diversified' investment sub-markets. To focus on this issue, we restate the Connor assumptions in the context of our two investor (domestic and foreign) clienteles[7]. For the finite factor economy,

(F1) There are μ^i investors, all of whom have risk-averse, von Neumann-Morgenstern utility functions, i={D,F}.

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7 With Ross and Walsh (1983), Solnik (1983) and Ikeda (1990), we assume that all currency risks are exactly spanned by factor portfolios.

(F2) There are $n^i \left(=n_i+n_{I_D}+n_{I_F}\right)$ risky assets with per-share payoffs obeying 3), and

$$V^i = E\left(\begin{bmatrix}\varepsilon_i(s,T)\\ \varepsilon_{I_D}(s,T)\\ \varepsilon_{I_F}(s,T)\end{bmatrix}\begin{bmatrix}\varepsilon_i(s,T)\\ \varepsilon_{I_D}(s,T)\\ \varepsilon_{I_F}(s,T)\end{bmatrix}'\right) \text{ and } E\left(f(s,T)f(s,T)'\right) \text{ exist for i=\{D,F\}.}$$

(F3) There exists a riskless asset with per-share payoff $1/p_o(t)$.

(F4) $$\left(\begin{bmatrix}\beta_i\\ \beta_{I_D}\\ \beta_{I_F}\end{bmatrix}'\begin{bmatrix}\beta_i\\ \beta_{I_D}\\ \beta_{I_F}\end{bmatrix}\right) \text{ and } E\left(\begin{bmatrix}e_i(s,T)\\ e_{I_D}(s,T)\\ e_{I_F}(s,T)\end{bmatrix}\begin{bmatrix}e_i(s,T)\\ e_{I_D}(s,T)\\ e_{I_F}(s,T)\end{bmatrix}'\right) \text{ are nonsingular for i=\{D,F\}.}$$

(F5) $$E\left(\begin{matrix}\varepsilon_i(s,T)\\ \varepsilon_{I_D}(s,T)\\ \varepsilon_{I_F}(s,T)\end{matrix}\middle| f(s,T)\right) = 0 \text{, for i=\{D,F\}}$$

(F6) Economy i has a local competitive equilibrium for i={D,F}.

For the limit economy,

(L1) There are μ^i investors, all of whom have risk-averse, von Neumann-Morgenstern utility functions, i={D,F}.

(L2) There are three countably infinite collections of risky assets, $j = \{D,\{I_D \cup I_F\},F\}$.

$$V^{N_i} = E\left(\begin{bmatrix}\varepsilon_i^{n_i}(s,T)\\ \varepsilon_{I_D}^{n_{I_D}}(s,T)\\ \varepsilon_{I_F}^{n_{I_F}}(s,T)\end{bmatrix}\begin{bmatrix}\varepsilon_i^{n_i}(s,T)\\ \varepsilon_{I_D}^{n_{I_D}}(s,T)\\ \varepsilon_{I_F}^{n_{I_F}}(s,T)\end{bmatrix}'\right) \text{ with } N_i = n_i + n_{I_D} + n_{I_F} \text{ for i=\{D,F\}.}$$

and $E\left(f(s,T)f(s,T)'\right)$ exist for every $\left\{n_D, n_{I_D}, n_{I_F}\right\}, \left\{n_F, n_{I_D}, n_{I_F}\right\}$.

(L3) There exists a riskless asset with per-share payoff $1/p_o(t)$.

(L4) $$\left(\begin{bmatrix}\beta_i^{n_i}\\ \beta_{I_D}^{n_{I_D}}\\ \beta_{I_F}^{n_{I_F}}\end{bmatrix}'\begin{bmatrix}\beta_i^{n_i}\\ \beta_{I_D}^{n_{I_D}}\\ \beta_{I_F}^{n_{I_F}}\end{bmatrix}\right) \text{ and } V^{N_i} \text{ are nonsingular for i=\{D,F\}.}$$

(L5) $$E\left(\begin{matrix}\varepsilon_i^{n_i}(s,T)\\ \varepsilon_{I_D}^{n_{I_D}}(s,T)\\ \varepsilon_{I_F}^{n_{I_F}}(s,T)\end{matrix}\middle| f(s,T)\right) = 0 \text{ i=\{D,F\}.}$$

Among Connor's four theorems, the third (linear pricing) and fourth (finite economy and limit economy isomorphism) theorems provide the basis for our pricing result. Equation 22) defines the factor pricing model in excess expected cashflow terms. Following Connor (1984), we restate this relation in terms of price:

$$\begin{bmatrix} p_i(t) \\ p_{I_D}(t) \\ p_{I_F}(t) \end{bmatrix} = p_o(t)\left(\begin{matrix} \alpha_i(t) \\ \alpha_{I_D}(t) \\ \alpha_{I_F}(t) \end{matrix} - \begin{bmatrix} \beta^{d_i} & \beta^{w_i} \\ 0 & \beta_{I_D} \\ 0 & \beta_{I_F} \end{bmatrix} \begin{bmatrix} \gamma^{d_i} \\ \gamma^{w} \end{bmatrix} \right) \qquad \text{for } i=\{D,F\} \qquad 26)$$

Theorem: With locally well-diversified portfolios,

1) The world factor prices (γ^w) are equal across investors and markets.

2) The domestic factor prices are not equal across investor country clienteles or markets $\left(\gamma^{d_D} \neq \gamma^{d_F}\right)$.

Proof: With locally well-diversified portfolios,

1) Given our factorization of all common factor risk into the world factors, the globally traded investments price the world factors. By Connor's (1984) Theorem 3, globally traded investments must trade for the same price accross domestic and foreign markets:

$$p_{I_i}^{D}(t) = p_o(t)\left(\alpha_{I_i}(t) - \begin{bmatrix} 0 & \beta_{I_i} \end{bmatrix} \begin{bmatrix} \gamma^{d_D} \\ \gamma^{w_D} \end{bmatrix} \right) = p_{I_i}^{F}(t) = p_o(t)\left(\alpha_{I_i}(t) - \begin{bmatrix} 0 & \beta_{I_i} \end{bmatrix} \begin{bmatrix} \gamma^{d_F} \\ \gamma^{w_F} \end{bmatrix} \right)$$

for i={D,F}. Therefore, $\gamma^{w_D} = \gamma^{w_F}$.

2) Consider the a) finite economy and b) limit economy cases:

a) In the finite economy case, assert that domestic market factor prices are equal: $\left(\gamma^{d_D} = \gamma^{d_F}\right)$. From section 3), our example provides a contradiction.

b) By Connor's Theorem 4 , a limit economy isomorphic to the finite factor economy exists. From case a), contradiction.

5. Conclusion

As long as international investors are free to arbitrage across a restricted set of international investments that embody common factors in their returns, the associated factor prices of risk must be equal across markets. However, common factor portfolios that are ‘uncommonly’ traded will have factor prices that are not necessarily equal.[8] This statement is true even if the restricted domestic investment factors are equivalent to the restricted foreign investment factors.

Without a mechanism to arbitrage across the restricted factor components of domestic and foreign investment returns, cross-country factor prices will differ (except by chance). Furthermore, the finite economy pricing relations that we have specifically identified will be indistinguishable from the parametrically equivalent ‘locally’ well-diversified limit economy case.

To re-emphasize the difference between the finite and limit economy cases, we quote Connor (1984 - p. 28): "In the finite case, asset supplies must be in a particular proportion such that idiosyncratic risks exactly cancel out of the market portfolio. ... In the limit case, ... Asset supplies need not be in any special ratio. Rather, the supply of each asset must be 'very small' (infinitesimal) on a per-capita basis." Clearly, the finite case serves as a useful construct for analyzing the characteristics and comparative statics of the more tenable and appealing limit factor economy case.

With regard to restricted international CAPM specifications, much extant empirical work has been based on complete market segmentation or market integration hypotheses. In many cases, the mixed result that multiple market indices are relevant for pricing investments across countries is found. To the extent that these test market index portfolios are recast as market factors instead of

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8 If the factors are not orthogonal, then the factor components idiosyncratic to the domestic and foreign markets will be priced differently. All common factor-related components will be priced equivalently across countries.

representative market portfolios, then the test results may be consistent with the partial market segmentation hypothesis.

Our results also indicate that any cross-country investments invalidate use of the domestic asset market portfolio as a reference pricing portfolio. Since domestic investment in foreign country investments must enter the domestic investors' reference pricing portfolio, market indices such as the S&P 500, FTSE 100, MSCI or World Bank emerging market country indices are not sufficiently broad to be identified as global reference pricing portfolios.

Linking locally well-diversified reference pricing portfolios with factor structure limits the degree of international investment pricing heterogeneity. Though the factor prices of risks that are uniquely embodied in restricted domestic and foreign investments will not be the same, the factor prices of risks that are embodied in globally traded securities must be equal across all investment sets.

Finally, our results shed some light on the country fund and futures (index) markets. The issuance of particular country funds and/or futures contracts can have significant impacts on expected equity returns. To the extent that country funds and futures change traded factor structure across countries, then both investment and factor prices will be affected.

Appendix: Identifying a Locally Well-Diversified Investment Cashflow Set

We specify two investor classes: domestic and foreign. These investors are aggregated within their 'home' countries and embody quadratic utility for two periods of consumption. The key development of our example is identification of a market clearing consistent with locally well-diversified investment portfolios.

The well-diversified set of investment returns is created conditional on an equally-weighted and joint Pareto-optimal expected utility optimization across both 'investors.'[9] In the optimization, the return innovations for the incremental investments that provide diversification are endogenous to the market clearing. This market clearing is defined as the optimum of the following weighed quadratic utility maximization problem:

$$\underset{\left\{q_o^D(t),q_{I_D}^D(t),q_{I_F}^D(t),\; p_o(t),p(t)\right\}}{\text{Max}} \sum_{i=\{D,F\}} 45 * c^i(t) - c^i(t)^2 + 45 * E\left[w^i(T)\right] - E\left[w^i(T)^2\right]$$

$$c^i(t) = w^i(t) - q^i(t)'\, p(t),\; w^i(T) = q^i(t)'\, x(s,t)$$

$$w^D(t) = 8.360,\; w^F(t) = 9.731$$

The necessary well-diversified error space is constructed from discrete and uncorrelated factor and return innovations. These joint-binominally distributed innovations are constructed by extension of the three-dimensional state variable system developed in Amin and Bodurtha (1995). The binomial seeds for the innovations are uncorrelated, mean zero and variance one. This innovation state space is the following:

Return\State	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
F_1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
I_1	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
I_2	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
D_1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
f^d	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
f^w	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1

9 This approach has also been used by Dumas (1989).

We define the first four rows of this innovation matrix to be the matrix ι.

To generate the return innovations, these binomial innovations are weighted by their standard deviations and the lower diagonal Cholesky decomposition matrix of the following correlation matrix:

Correlation Matrix							
Return	F_1	I_1	I_2	D_1	f^d	f^w	Standard deviations
F_1	1	0.4	0.3	0.4	0	0	0.8
I_1	0.4	1	0.2	0.1	0	0	0.6
I_2	0.3	0.2	1	0.5	0	0	0.2
D_1	0.4	0.1	0.5	1	0	0	0.8
f^d	0	0	0	0	1	0	1.0
f^w	0	0	0	0	0	1	1.0

We define the correlation matrix of the four investments (the first four rows and columns) to be R, and a diagonal matrix with the four associated standard deviations along the diagonal to be S. Since the correlation matrix R is positive definite, we may define its Cholesky decomposition: R=P'P. We calculate the four by 16 investment return innovation matrix, ε, as the following:

$$\varepsilon = P'S'\iota$$

In the discrete finite factor economy case, the existence of locally well-diversified portfolios requires that the two remaining local market errors be linearly dependent on the other errors. Specifically, these errors will be related to the domestic and foreign aggregate investors' portfolio allocations:

$$\varepsilon_{D_2} = -\left(\varepsilon_{D_1} * q^{D}_{D_1} + \varepsilon_{I_1} * q^{D}_{I_1} + \varepsilon_{I_2} * q^{D}_{I_2}\right),\ \varepsilon_{F_2} = -\left(\varepsilon_{F_1} * q^{F}_{F_1} + \varepsilon_{I_1} * q^{F}_{I_1} + \varepsilon_{I_2} * q^{F}_{I_2}\right)$$

The investment cashflows are defined over the equiprobable states, s = {1,...,16}:

$$x_j(s,T) = \alpha_j(t) + \beta_j f(s,T) + \varepsilon_j(s,T),\quad j = (F_1,F_2,I_1,I_2,D_1,D_2),$$

Since two of the investment return innovations are dependent on investor allocations, identification of a market-clearing allocation and associated investment prices requires solution of a recursive and, hence, nonlinear optimization problem. We solve this problem numerically with the GAUSS OPTMUM routine.

The resulting cashflow set satisfies the undiversified and well-diversified pricing equations 19) and 23), respectively. Equivalently, the standard Shanken (1985) two-stage factor price regression recovers the specified parameters.

If one wishes to test partially restricted factor pricing models in this standard cross-section regression context, then a linearized three-stage procedure (following Gibbons (1982)) will be necessary. Due to the more complicated errors-in-variable characteristics for a restricted factor pricing model test, the associated two-stage procedure, which is appropriate for the integrated or segmented market cases, is misspecified. The maximum likelihood procedure of Jorion and Schwartz (1986) extends directly.

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Table 1: International Financial Market Integration-Related Empirical Research

Authors (year)	Countries	Time-Series	Specification	Findings
Solnik (1974)	Be, Fr, Ge, It, Ja, Sd, Sw, UK	Eurofinance 234 European and 65 U.S. stocks: 3/66-4/71 weekly	Unconditional CAPM	Evidence of pricing both domestic and world beta risk
Stehle (1977)	US and World index of Be, Ca, Fr, Ge, It, Ja, Ne, Sw, and UK	MSCI for world index components and CRSP for US index: 1/56-1/75 monthly	Unconditional CAPM	Neither US segmented market nor World integrated market rejected
Errunza-Losq (1985)	Ar, Br, Ch, Gr, In, Ko, Me, Th, US, Zi	World bank country indices, US CRSP index, constructing both GNP- and market value-weighted world portfolios: monthly 1/76-12/80	Unconditional CAPM	No significant rejection of integration or segmentation
Jorion-Schwartz (1986)	Ca and US	1040 Canadian stocks sorted into nine domestic and global beta-sorted portfolios: 6/63-12/82 monthly	Unconditional CAPM	<u>Segmentation with residual Canadian market risk being significantly priced</u>
Cho-Eun-Senbet (1986)	Al, Ca, Fr, Ge, HK, Ja, Ne, Si, Sw, UK, US	349 MSCI stocks 1/73-12/83 monthly	Unconditional (cross-country return inter-battery) factor loading portfolios across 55 country pairs	<u>Equality factor risk premium estimates is rejected for of 30 out of 55 country pairs</u>
Wheatley (1988)	Al, Au, Be, Ca, De, Fr, Ge, HK, It, Ja, Ne, No, Si, Sp, Sd, Sw, UK, US	Ibbotson-Sinquefield 1 month T-bill, Gov't bond, corp. bond; CRSP 20 U.S. size-ranked portfolios, a US ADR portfolio, 17 MSCI indices, US consumption, IFS spot and FT forward currency prices: 1/60-12/85 monthly	Unconditional consumption CAPM	No segmentation

Table 1 (continued)

Authors (year)	Countries	Time-Series	Specification	Findings
Bodurtha-Cho-Senbet (1989)	Al, Ca, Fr, Ge, Ja, UK, US	263 MSCI stocks and IMF-OECD macro factors: 1/73-12/83 monthly	Unconditional (return & macro factor inter-battery) factor loading portfolios across 6 countries paired with the U.S.	Risk premia estimate equality maintained for all country pairs except Japan and US
Bonser-Neal, Bauer, Neal, Wheatley (1989)	Fr, Ja, Ko, Me, Ta	CRSP, Barron's Country fund premia over associated NAV portfolio: 5/81-1/89 weekly	Dummy variable intervention analysis of investment restriction regulation changes	Four of five premia significantly differ before and after the regulation change
Gultekin,-Gultekin-Penati (1989)	Ja, US	Industry-matched sample of 110 Japanase and 110 US firms: 1/77-12/84 weekly (Wednesdays)	Unconditional factor loading portfolios - structural break with Japanese regulation change 12/80-1/81	Segmented zero-beta and factor prices before 1981, and no significant difference subsequently.
Giovanni-Jorion (1989)	BP, DM, SF deposits and U.S. equity index	DRI daily currency, 1 week FT Euro rates, CRSP index and Treasury investment data,: 7/74-12/86 weekly	Conditional CAPM - market price of risk and GARCH(1,1)-X (X is lagged local rate innovation) investment allocation-weighted conditional covariance	Reject CAPM specification
Hietala (1989)	Fi, world	25 Finnish restricted and unrestricted stocks and MSCI world index: 1/79-12/83 monthly	Unconditional CAPM and beta risk-adjusted price perpetuity growth model	25% price premium estimate for the restricted stocks

Table 1 (continued)

Authors (year)	Countries	Time-Series	Specification	Findings
Korajczyk-Viallet (1989)	Fr, Ja, UK, US	4211-6692 stock returns to generate factors and form four country specific sets and one common set of size-ranked decile portfolios; Ibbotson T-bill and inflation, and DRI currency: 1/69-12/83 monthly	Time-varying factor portfolio APT and single market factor (CAPM) with ummy variable intervention analysis of 1974 and 1979 investment restriction regulation changes	1974 dummy significant under APT (not for CAPM) and 1979 dummy not significant except for France
Cumby (1990)	Ge, Ja, UK, US	MSCI returns: 1/74-12/87 quarterly	Conditional (latent variable and constant beta ratio) consumption CAPM	Mild rejecction of model over whole sample, but not rejected for 1980 sub-sample
Cumby-Glen (1990)	Al, Be, Ca, Fr, HK, It, Ja, Ne, Si Sw, UK, US and US int'l mutual funds	MSCI returns: 1/82-6/88 monthly	Unconditional CAPM	Maintain integration
Harvey (1991)	Al, Au, Be, De, Fi, Fr, Ge, HK, It, Ja, Ma/Si, Ne, No, NZ, Sp, Sd, Sw, UK, US, world	MSCI indices: 12/69-5/89 monthly	Conditional CAPM - market price of risk	Differing estimated prices or risk
Chan-Karolyi-Stulz (1992)	Ja, US, EAFE	Nikkei 225 and CRSP: 1/78-12/89monthly, and MSCPI Japan and EAFE: 1/80-12/89 monthly	Conditional CAPM - market price of risk and GARCH(1,1) investment allocation-weighted conditional covariance	Integration not rejected

Table 1 (continued)

Authors (year)	Countries	Time-Series	Specification	Findings
Errunza-Losq (1992)	Ar, Br, Cl, Gr, In, Ko, Me, Zi, US	833 US CRSP formed into 36 US and world beta-weighted portfolios, and 381 EMDB stocks, IFS currency: monthly 12/75-12/87	Unconditional CAPM	Integration rejected, complete segmentation rejected except Ar and Zi; mild segmentation maintained except India
Bansal-Hsieh-Viswanathan (1993)	Ge, Ja, UK, US and world	1 week Eurocurreny deposit returns, 4 week US T-bill and forward currency returns, MSCI stock indices, : 1/75-12/90 weekly	Integrated markets unconditional linear, conditional linear and Unconditional nonlinear APT tests by general GMM metric and Hansen-Jagannathan (1991) distance metrci	Equity models not rejected, unconditional linear model rejected for interest rate and currency returns, and unconditional nonlinear model performs "best"
Bailey-Jagtiani (1994)	Th, US, world	27 Main and Alien Board traded stocks, MSCI world index, SET index, Bhat, information set (trade-weighted dollar value, change in US and Thai rates, dividend yields and rate term premia): 1/88-12/92 monthly	Unconditional CAPM for differences of Main and Alien Board stock returns	Thai A and B Board share markets are segmented
Brown-Otsuki (1994)	Al, Au, Be, Ca, Fr, Ge, HK, Ir, It, Ja, Ma, Ne, No, NZ, SA, Sp, Sd, Sw, UK, US	Ibbotson - instruments and factors (Japan yield spread, US yield spread, US inflation and US dividend yield), residual factors (US market, US small stocks, UK s/t bill return, yen return)	Conditional integrated market APT	General model specification maintained

Table 1 (continued)

Authors (year)	Countries	Time-Series	Specification	Findings
Cooper-Kaplanis (1994)	Fr, It, Ge, Ja, Sd, Sp, UK, US	LSPD equity indices and IFS price indices: 1/78-12/87 monthly	Unconditional CAPM associated estimates of implicit investment restriction costs for aggregate investor holdings	Model specification is rejected and, given conventional levels of risk aversion the estimated costs too high
Bekaert-Harvey (1995)	Cl, Co, Gr, In, Jo, Ko, Ma, Me, Ni, Ta, Th, Zi	MSCI developed country: 1/70-12/92 monthly and IFC emerging country : 1/76-12/92 monthly	Conditional CAPM - market price(s) of risk and GARCH(1,1) investment allocation-weighted conditional covariance with conditional regime switching (more/less restricted) transitions by logistic probability	Significant integrated-segmented regime switching probability estimates
Bodurtha-Kim-Lee (1995)	Al, Fr, Ge, India, It, Ko, Ma, Me, Sp, Sw, Ta, Th, UK, US	13 country funds and associated market index NAV, US CRSP and currency exchange rates: 3/70-12/91 monthly	Unconditional factor pricing	Country fund and associated NAV portfolio "betas" differ significantly implying segmentation
Dumas-Solnik (1995)	Ge, Ja, US, UK, world	Local currency and equity index portfolios: 3/70-12/91 monthly	Conditional CAPM - market price of risk augmented with currency deposit prices of risk	Currency and equity portfolio integration maintained

Table 1 (continued)

Authors (year)	Countries	Time-Series	Specification	Findings
Harvey (1995)	Emerging (Ar, Br, Cl, Co, Gr, Id, In, Ja, Jo, Ko, Ma, Me, Ni, Pa, Po, Ta, Th, Ve, Zi), Developed (Al, Au, Be, De, Fi, Fr, Ge, HK, It, Ja, Ne, No, NZ, Sp, Sd, Sw, UK, US, world)	EMDB shares and indices, MSCI (developed) indices and Datastream G-10 FX and Eurorates : 1/76-12/92 monthly	Conditional CAPM and unconditional CAPM with global and local information	Integrated unconditional model rejected and significance of local information suggests segmentation in the conditional tests.
Stulz-Wasserfallen (1995)	Sw, world	19 Swss dual share (unrestricted/restricted) class firms, MSCI world index: 1/85-12/89 weekly	Price discriminating restricted and unrestricted share issuer to mean-variance optimizing investors	"world betas don't matter ... indicating segmentation"
Bekaert-Urias (1996)	Country funds (Au, Fr, Ge, Ir, It, Ja, Si, Sw, UK), IFC investible indices (Ar, Br, Cl, In, Id, Ma, Me, Ph, Po, Ta, Th, Tu) UK, US	37 UK and 43 US country funds of which 23 and 19 are, respectivly emerging market invested, and 12 IFC investible indices; fund data from FT, Bloomberg and security firms	Conditional test of incremental gains to broader investment by Hansen-Jagannathan (1991) and Huberman-Kandel (1987) test variant	Incremental benefits to UK country fund and IFC investible portfolios
Domowitz, Glen-Madhaven (1997)	24 (daily) and 46 (weekly) Me dual class equity return series for 21 frims: 1/90-12/93.	MSCI indices: 1/70-12/94 monthly	Premium modeled as function of % of unrestricted shares, % volume of unrestricted shares, company value, dividend yield, and currency risk.	Significant price premium for restricted shares and related to % of unrestricted shares, company value and currency risk

Table 1 (continued)

Authors (year)	Countries	Time-Series	Specification	Findings
DeSantis-Gerard (1997)	Ca, Fr, Ge, It, Ja, Sw, UK, US	MSCI indices: 1/70-12/94 monthly	Conditional CAPM - market price(s) of risk and GARCH(1,1) investment allocation-weighted conditional covariances	Integration not rejected, but significant time-varying US investor benefits to international diversification documented
Su (1999)	Ch and US	47 A and B share traded Chinese firms, Hang Seng, MSCI world, NASDAQ and NYSE indices: weekly 4/94-9/96.	Unconditional CAPM for differences of A and B share stock returns	Chinese A and B share markets are segmented

Country codes: Al-Australia, Ar-Argentina, Au-Austria, Be-Belgium, Bo-Bolivia, Br-Brazil, Ca-Canada, Ch-China, Cl-Chile, Colombia-Co, De-Denmark, Fi-Finland, Fr-France, Ge-Germany, Gr-Greece, HK-Hong Kong, Id-Indonesia, In-India, Ir-Ireland, It-Italy, Ja-Japan, Jo-Jordan, Ko-South Korea, Ma, Malaysia, Me-Mexico, Ne-Netherlands, NZ-New Zealand, Ni-Nigeria, No-Norway, Pa-Pakistan, Ph-Phillipines, Po-Portugal, Sd-Sweden, Si-Singapore, SA-South Africa, Sp-Spain, Sw-Switzerland, Ta-Taiwan, Th-Thailand, Tu-Turkey, Ve-Venezuala, UK-United Kingdom, US-United States, Zi-Zimbabwe.

Data sources: EMDB - World Bank Emerging Markets Data Bank, FT - Financial Times, IFC - International Finance Corp., LSPC - London Share Price Database, MSCI - Morgan Stanley Capital International.

Table 2: Investment Cashflow Definitions and Components

$x_j(s,T) = \alpha_j(t) + \beta_j f(s,T) + \varepsilon_j(s,T)$, $j = (F_1,F_2,I_1,I_2,D_1,D_2)$, equiprobable states s = {1,...,16}

Investment Cashflow	Riskless	Foreign F_1	Foreign F_2	International I_1	International I_2	Domestic D_1	Domestic D_2	Factors f^d	Factors f^w
Expected (α)	1.0	10.0	5.0	6.0	3.0	4.0	6.0	0.0	0.0

Innovations (ε, f) in State (s)	Riskless	Foreign F_1	Foreign F_2	International I_1	International I_2	Domestic D_1	Domestic D_2	Factor f^d	Factor f^w
1	0.00	0.80	-1.36	0.79	0.27	1.25	-1.75	1.00	1.00
2	0.00	0.80	-1.36	0.79	0.27	-0.06	-0.44	-1.00	-1.00
3	0.00	0.80	-0.57	-0.31	-0.15	0.70	-0.48	1.00	-1.00
4	0.00	0.80	-0.57	-0.31	-0.15	-0.61	0.83	-1.00	1.00
5	0.00	-0.80	0.67	0.31	-0.23	-0.04	0.09	-1.00	1.00
6	0.00	-0.80	0.67	0.31	-0.23	-1.35	1.40	1.00	-1.00
7	0.00	-0.80	1.25	-0.79	0.11	0.71	-0.49	-1.00	-1.00
8	0.00	-0.80	1.25	-0.79	0.11	-0.60	0.82	1.00	1.00
9	0.00	-0.80	1.36	-0.79	-0.27	-1.25	1.75	1.00	1.00
10	0.00	-0.80	1.36	-0.79	-0.27	0.06	0.44	-1.00	-1.00
11	0.00	-0.80	0.57	0.31	0.15	-0.70	0.48	1.00	-1.00
12	0.00	-0.80	0.57	0.31	0.15	0.61	-0.83	-1.00	1.00
13	0.00	0.80	-0.67	-0.31	0.23	0.04	-0.09	-1.00	1.00
14	0.00	0.80	-0.67	-0.31	0.23	1.35	-1.40	1.00	-1.00
15	0.00	0.80	-1.25	0.79	-0.11	-0.71	0.49	-1.00	-1.00
16	0.00	0.80	-1.25	0.79	-0.11	0.60	-0.82	1.00	1.00

Factor Betas (β)	Riskless	Foreign F_1	Foreign F_2	International I_1	International I_2	Domestic D_1	Domestic D_2	Factors f^d	Factors f^w
Domestic	0.0	1.4	0.0	0.0	0.0	1.4	0.0	1.0	0.0
World	0.0	0.0	0.0	1.1	0.7	0.0	0.0	0.0	1.0

Table 3: Investment Cashflow [x(s,T)] Covariance Matrix

		Riskless	Foreign F_1	Foreign F_2	International I_1	International I_2	Domestic D_1	Domestic D_2	Factors f^d	Factors f^w
Riskless		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Foreign	F_1	0.00	2.60	-0.77	0.19	0.05	2.22	-0.36	1.40	0.00
	F_2	0.00	-0.77	1.05	-0.42	-0.07	-0.31	0.52	0.00	0.00
International	I_1	0.00	0.19	-0.42	1.57	0.79	0.05	-0.20	0.00	1.10
	I_2	0.00	0.05	-0.07	0.79	0.53	0.08	-0.12	0.00	0.70
Domestic	D_1	0.00	2.22	-0.31	0.05	0.08	2.60	-0.72	1.40	0.00
	D_2	0.00	-0.36	0.52	-0.20	-0.12	-0.72	0.88	0.00	0.00
Factor	f^d	0.00	1.40	0.00	0.00	0.00	1.40	0.00	1.00	0.00
	f^w	0.00	0.00	0.00	1.10	0.70	0.00	0.00	0.00	1.00

Symbols are defined in the Investor's Decision Problem section 1.1) of the paper.

Table 4: Investor Attributes and Market-Clearing Values

Panel A: Domestic and foreign investors have two period quadratic utility and following attributes:

$$\max_{\{q_o^D(t),\, q_{I_D}^D(t),\, q_{I_F}^D(t),\, p_o(t),\, p(t)\}} \sum_{i=\{D,F\}} 45 * c^i(t) - c^i(t)^2 + 45 * E\left[w^i(T)\right] - E\left[w^i(T)^2\right]$$

$w^D(t) = 8.360$, $w^F(t) = 9.731$, $c^i(t) = w^i(t) - q^i(t)'p(t)$, $w^i(T) = q^i(t)'x(s,T)$

Panel B: Undiversified Economy

Allocation	Investment	Riskless q_o	Foreign q_{F_1}	q_{F_2}	International q_{I_1}	q_{I_2}	Domestic q_{D_1}	q_{D_2}
Initial	Domestic $q^D(t)$	0.5	0.0	0.0	0.5	0.5	1.0	0.0
	Foreign $q^F(t)$	0.5	1.0	0.0	0.5	0.5	0.0	0.0
Market clearing	Domestic $q^D(t)$	2.88	0.0	0.0	0.78	0.12	1.0	0.0
	Foreign $q^F(t)$	-1.88	1.0	0.0	0.22	0.88	0.0	0.0

Panel C: Well-Diversified Economy

Allocations	Investment	Riskless q_o	Foreign q_{F_1}	q_{F_2}	International q_{I_1}	q_{I_2}	Domestic q_{D_1}	q_{D_2}
Initial	Domestic $q^D(t)$	0.5	0.0	0.0	0.5	0.5	1.0	1.0
	Foreign $q^F(t)$	0.5	1.0	1.0	0.5	0.5	0.0	0.0
Market clearing	Domestic $q^D(t)$	2.83	0.0	0.0	0.38	0.73	1.0	1.0
	Foreign $q^F(t)$	-1.83	1.0	1.0	0.61	0.27	0.0	0.0

Panel D: Undiversified and Well-Diversified Economy Investment Prices
(shadow prices of restricted investments are in *italics*)

Investment Values		Riskless p_o	Foreign p_{F_1}	p_{F_2}	International p_{I_1}	p_{I_2}	Domestic p_{D_1}	p_{D_2}	Factor costs f^d	f^w
Undiversified	Domestic	0.164	*1.396*	n/a	0.843	0.413	0.382	n/a	-0.146	-0.098
	Foreign	0.164	1.333	n/a	0.843	0.413	*0.392*	n/a	-0.162	-0.099
Diversified	Domestic	0.276	*2.654*	*1.379*	1.600	0.793	0.999	*1.655*	-0.075	-0.050
	Foreign	0.276	2.646	1.379	1.600	0.793	*0.991*	1.655	-0.080	-0.050

Panel E: Prices of Risk

	Investments	λ	γ^d	γ^w	γ^{ε_p}
Undiversified	Domestic	0.634	0.888	0.598	0.608
	Foreign	0.702	0.984	0.602	0.609
Well-diversified	Domestic	0.193	0.270	0.180	n/a
	Foreign	0.207	0.290	0.180	n/a