

# Historical and Implied Measures of “Value at Risk”: The DM and Yen Case

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## Abstract

“Value at risk” is a measure of corporations’ and banking institutions’ sensitivity to market price movements. This measure has been adopted as a global capital adequacy standard. Since value at risk management is a portfolio application, value at risk measurement requires standard deviation and correlation estimates. Following price value at risk convention, we analyze ex-post dollar/mark and dollar/yen log exchange rate change sample standard deviations and dollar/mark-dollar/yen log exchange rate change sample correlations. To examine the information content of ex-ante standard deviation and correlation estimates, we use historical based forecasts, as well as option implied forecasts

Among historical forecasts, we examine simple lagged monthly sample statistics, a proposed weighted moving average alternative, and a rolling bivariate GARCH(1,1) forecast. The option implied covariance parameter forecasts are derived from the unique Philadelphia Stock Exchange-traded currency and cross-currency options: dollar/mark, dollar/yen and yen/mark. We document very strong auto-correlation in the standard deviation and correlation estimates. Faced with the possibility of integrated variables in our forecasting regressions, we use the robust Stock-Watson (1993) estimation method. Among the evaluated forecast methods, only the GARCH(1,1)-based estimator fails to indicate significance.

The historical and implied covariance dynamics suggest specific concern with "covariance value at risk." In this context, we document a positive correlation between changes in mark and yen implied volatility estimates and yen/mark implied correlation estimates. This relationship indicates the pricing of increased covariance value at risk (less dollar-denominated diversification) as dollar/mark and dollar/yen option prices increase.

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### Abstract

“Value at risk” is a measure of corporations’ and banking institutions’ sensitivity to market *price* movements. This measure has been adopted as a global capital adequacy standard. Since value at risk management is a portfolio application, value at risk measurement requires standard deviation and correlation estimates. Following price value at risk convention, we analyze ex-post dollar/mark and dollar/yen log exchange rate change sample standard deviations and dollar/mark-dollar/yen log exchange rate change sample correlations. To examine the information content of ex-ante standard deviation and correlation estimates, we use historical based forecasts, as well as option implied forecasts.

Among historical forecasts, we examine simple lagged monthly sample statistics, a proposed weighted moving average alternative, and a rolling bivariate GARCH(1,1) forecast. The option implied covariance parameter forecasts are derived from the unique Philadelphia Stock Exchange-traded currency and cross-currency options: dollar/mark, dollar/yen and yen/mark. We document very strong auto-correlation in the standard deviation and correlation estimates. Faced with the possibility of integrated variables in our forecasting regressions, we use the robust Stock-Watson (1993) estimation method. Among the evaluated forecast methods, only the GARCH(1,1)-based estimator fails to indicate significance.

The potential non-stationary aspect of historical and implied covariance dynamics suggests specific concern with “covariance value at risk.” In this context, we document a positive correlation between changes in mark and yen implied volatility estimates and yen/mark implied correlation estimates. This relationship indicates the pricing of increased covariance value at risk (less dollar-denominated diversification) as dollar/mark and dollar/yen option prices increase.

The measurement and management of market price risk has become a significant concern for regulators, commercial and investment banks, corporations and institutional investors.<sup>1</sup> “Market risk involves uncertainty of earnings resulting from changes in market

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<sup>1</sup> Group of Thirty (1993), General Accounting Office (1994), Bank for International Settlements (1994).

conditions (price of the asset, interest rates, volatility, market liquidity).”<sup>2</sup> “Value at risk” is a measure that can “quantify market risks in portfolios of fixed income and equity positions.”<sup>3</sup>

Most value at risk measures are based on price change standard deviations and correlations. The underlying assumption of this analysis is that these price changes or log price changes are approximately normally distributed. To treat price risks that are clearly non-normal such as derivatives, these risks have been represented by either linear (delta-based) functions of market price risks or by analyzing changes in univariate measures of derivative value, such as implied volatility.

In this work, we analyze dollar-denominated German mark and Japanese yen value at risk. We focus on the mark and yen for three reasons. First, because the mark and yen float relatively freely against the dollar, we can use standard European and American currency option pricing models. Second, combined yen and mark currency trade comprises over half of all foreign exchange market activity in London, New York and Tokyo.<sup>4</sup> Third, the late 1991 Philadelphia Stock Exchange (PHLX) listing of yen/mark cross-currency options completed the implicit two dimensional yen/mark covariance parameter space.

With dollar/mark and dollar/yen options trading and implicitly pricing mark and yen volatility since 1982, the trading of yen/mark options implicitly prices the dollar/mark and dollar/yen exchange rate correlation. Estimates of implied volatility and correlation parameters provide ex-ante assessments of the components of dollar/mark and dollar/yen

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<sup>2</sup> JP Morgan (1995).

<sup>3</sup> JP Morgan (1994).

<sup>4</sup> Bank of England (1989).

value at risk. To date, the PHLX currency options are the only active exchange-traded options which embody correlation pricing.

We evaluate implied covariance estimates and three x-ante measures of historical covariance as predictors of subsequent (ex-post) mark and yen value at risk. The historical-based measures are: 1) simple lagged sample standard deviations and correlations, 2) specific exponentially-weighted moving average alternatives which have been proposed by JP Morgan-Riskmetrics, and a 300-day rolling bivariate GARCH(1,1) estimator.<sup>5</sup> Among these estimators, the GARCH(1,1) estimator is relatively problematic and computationally intensive in the forecasting context.

To determine the yen/mark option implied covariance matrix, we use PHLX dollar/mark, dollar/yen and yen/mark currency option trade prices to imply the yen/mark covariance matrix. From this covariance matrix, we calculate implied volatilities (standard deviations) and correlations.

In the empirical derivatives literature, the regression of average squared currency price innovations on implied volatility and time series-based variance forecasts is a standard test for predictability. In this study, we extend this method to treat correlations. Our prediction tests are based on regressions using 30-day and 60-day forecast horizons. In practice, monthly horizons have been a standard interval for measuring “value at risk”.<sup>6</sup>

We regress leading 30-day and 60-day measures of covariation on both the associated implied parameter and the three historical forecasts. Effectively, we test if future standard

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<sup>5</sup> JP Morgan (1994), pp. 51-66. In choosing the 300 day GARCH forecasting window, we follow Lamoureux and Lastrapes (1993)

deviations and correlations are related to the associated time series and/or implied measures of standard deviation and correlation.

Our regression results indicate that lagged sample standard deviations and correlations, exponentially weighted moving average alternatives, and option implied parameter-related variables are related to future levels of volatility and correlation. The rolling bivariate GARCH(1,1)-based forecasts have limited explanatory power, and in most cases the associated forecast regression coefficients are of the wrong sign (negative.) Finally, the levels of explanatory power for most of the forecasting equations are, generally, modest. ( $R^2 \in [0.05, 0.49]$ ).<sup>7</sup>

Our finding that option implied yen/mark correlations are linked to future levels of yen/mark cross-exchange rate correlation extends previous research that documents a relationship between future volatility and current implied volatility.<sup>8</sup> That is, we find that implied correlations provide explanatory power similar to that of implied volatilities.

As a by-product of our forecasting analysis, we note that most of our time series are close to being integrated (having unit roots.) For the historical estimates, a good part of this phenomenon may be attributed to the overlapping squared errors that enter the daily lagged calculations. Nevertheless, the implied volatility estimate also manifests strong persistence. Facing potentially integrated regressors and regressands, we have used the Stock-Watson (1993) estimation procedure.

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<sup>6</sup> JP Morgan (1995), p. 2.

<sup>7</sup> Canina and Figlewski (1993) emphasize the lack of fit in their regressions of ex-post volatility on ex-ante implied volatility estimates.

<sup>8</sup> See Akgiray (1989), Day and Lewis (1992) and Lamoureux and Lastrapes (1993).

Finally, we examine the implied volatility and correlation time series as potential measures of "covariance value at risk." This issue is an increasing concern for risk managers, and has been manifest in the historically positive relationship between the volatilities and correlations of international equity and bond markets.<sup>9</sup> Such a relation is problematic for risk managers because periods of high risk (high standard deviations) are associated with lower diversification opportunities (higher correlations).

Indeed, we document an analogous implied yen/mark foreign exchange rate risk phenomenon. That is, higher mark and yen implied volatilities are associated with higher yen/mark implied correlations. Furthermore, this currency option market phenomenon indicates the actual *pricing* of a positive relation between volatility and correlation.

## **1. Data**

For our study, we use three data sources. First, we use the PHLX matched option trade and simultaneous Telerate spot currency bid-ask prices to value the dollar/mark, dollar/yen and yen/mark options. The PHLX database also provides option trading volume, but does not report option bid and ask prices. Second, we obtain dollar, mark and yen short-term Eurorates and long-term swap rates from Datastream. The overnight, one week, one month, three month, six month, one year, two year and three year rates are used to interpolate discount rates that are then matched to the option maturities. Finally, we use Datastream currency prices to create leading and lagging currency spot price innovation variables. These prices are quoted daily in New York by Bankers' Trust.

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<sup>9</sup> See Longin and Solnik (1991) and King, Sentana and Wadhvani (1994). For mark and British pound currencies, see Bollerslev and Engle (1993).

## 2. Forecast Regression Variables

Our forecast regressions require ex-post or leading standard deviation and correlation dependent variables and pre-determined or lagging predictor variables. For standard deviations, we regress leading 30-day and 60-day squared spot currency price innovations on four predetermined regressors: respective lagged 30-day and 60-day historical sample standard deviations, lagged 151-day exponentially declining weighted moving average estimates, rolling bivariate GARCH(1,1) estimates, and/or implied volatilities. Analogously, leading 30-day or 60-day correlations are regressed on lagged implied correlations, lagged 30-day or 60-day sample correlations, weighted moving average-based correlation estimates, and bivariate GARCH(1,1) estimates.

Table 1 documents summary statistics for the 30-day squared spot currency price innovations. The sample characteristics of the 60-day innovations are similar. Three important features of the estimated parameters are apparent. First, the scale of the standard deviations is on the order of 0.1 or 10%. These estimators have been annualized based on a 262 business day year. With regard to scale, the mark/yen correlation is around 0.5. Second, the estimates are both skewed and manifest non-normal tails. Finally, the estimates are very highly auto-correlated. A major part of this autocorrelation occurs by construction: each day the 30-day estimator shares 29 days of squared price change errors with the previous day estimator. Analogously, the 60-day estimator shares 59 days of observations from one day to the next.

Among our forecasting variables, the simple lagged estimators and the JP Morgan Riskmetrics-related exponentially weighted moving error estimators are most straightforward. Specifically, the variable definitions are the following:

$$\begin{aligned} \text{Historical} \quad \sigma_{t,k}^h &= \sqrt{\sum_{j=1}^{i_t} \varepsilon_{t+j,k}^2 / (i_t - 1)}, k = \text{DM or JY} \\ \text{Historical} \quad \rho_{t,\text{DM},\text{JY}}^h &= \sum_{j=1}^{i_t} \varepsilon_{t+j,\text{DM}} \varepsilon_{t+j,\text{JY}} / (i_t - 1) / \left( \sigma_{t,\text{DM}}^h \sigma_{t,\text{JY}}^h \right) \\ \text{Weighted} \\ \text{Historical} \\ (\lambda=0.97) \quad \sigma_{t,k}^w &= (1-\lambda) \sum_{j=0}^{151} \lambda^j \varepsilon_{t-j+n,k}^2, \lambda = 0.97, k = \text{DM or JY} \\ \text{Weighted} \\ \text{Historical} \\ (\lambda=0.97) \quad \rho_{t,\text{DM},\text{JY}}^w &= (1-\lambda) \sum_{j=0}^{151} \lambda^j \varepsilon_{t-j+n,\text{DM}} \varepsilon_{t-j+n,\text{JY}}, \lambda = 0.97 \\ \varepsilon_{t,k} &= \ln(S_t/S_{t-h}) - \mu_k h_t; h_t \text{ is the time between observations.} \\ i_t &\text{ is the number of trading days in the 30 or 60 days subsequent to date } t. \end{aligned}$$

As the historical estimators are simply the 30 and 60 day lags of the forecast variables, the Table 1 summary is applicable to these variables also. Table 2 reports the sample statistics for the exponentially weighted estimators. These estimators have similar sampling properties to the simple lagged estimators. However, the autocorrelation and Dicky-Fuller statistics for these estimators are even more significant than in the previous case.

Figure 1 provides plots of the two squared error-based time series. We see that "realized 30 day" squared error-based estimator is more variable than the exponentially smoothed "Riskmetrics" estimator. No strong trends appear to exist in the time series, and this observation suggests that evidence of unit root properties is not related to trends.

## 2.1 Rolling Bivariate GARCH(1,1) Predictors

Estimation of a set of out-of-sample generalized auto-correlated and conditionally heteroskedastic (GARCH) covariance forecasts is not trivial. We follow Lamoureux and

Lastrapes (1993), and use 300 observations to estimate our covariance matrix forecasts. A major problem for this exercise is enforcing a positive definite covariance matrix estimate for each forecast date, and in rolling the estimation equation forward 30 or 60 days so that the forecast covariance matrix remains positive definite over the 30 or 60 day forecast horizon.

Any such estimation exercise will, by necessity, be somewhat arbitrary. To fully define our estimation procedure, we start with our equation system definition:

$$S_t = \begin{bmatrix} \ln(S_{DMt}/S_{DMt-1}) \\ \ln(S_{JYt}/S_{JYt-1}) \end{bmatrix} = \mu + \varepsilon_t, \mu = \begin{bmatrix} \mu_{DM} \\ \mu_{JY} \end{bmatrix}, \varepsilon_t = \begin{bmatrix} \varepsilon_{DMt} \\ \varepsilon_{JYt} \end{bmatrix} \sim N(0, H_t)$$

$$H_t = \begin{bmatrix} a_{11} & 0 \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \varepsilon_t \varepsilon_t' \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} H_{t-1} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

This BEKK multivariate-GARCH specification of Engle-Kroner (1995) ensures that the covariance estimator will be positive semi-definite. We estimate the parameters by maximum likelihood. As noted, we not only require the covariance matrix estimator to be positive definite but also the forecasts. Therefore, we introduce likelihood function penalties to the following measures of estimated and forecast covariance matrix singularity: the determinant is too close to zero, the correlation is too close to one, and the variance of either series grows too large.

Unfortunately, this estimation exercise is so ill-conditioned and computationally intensive that these simple penalty function "fixes" are not sufficient to generate estimates in real or any reasonable time. Therefore, we also introduced an estimation regularizer that effectively penalizes shifts in parameter estimates from day to day or from an unconditional set of starting values. Generally, the associated regularizer weights were quite small. However for a few very problematic days, it was necessary to, effectively, set the associated

regularizer weight to infinity and simply use a previous days estimates. This relatively formal (but still heuristic) procedure roughly mimics what risk management professionals do in practice.

Table 3 reports the results of GARCH(1,1) estimation over the sample period and the average of the rolling coefficient estimates. For the November 1991-February 1994 sample period, the coefficients associated with the lagged variance (c's) are each insignificant. Despite this finding, including the lagged variance term in the GARCH specification did significantly add to model fit.

The averages of forecast equation coefficients across all 590 separate forecasts are very different from the "in sample estimates." Furthermore, these estimates are, effectively, an ordered "boot-strap" set of estimates drawn over a September 28, 1990 - February 24, 1994 sample period. (September 28, 1990 is the first observation entering the November 21, 1991 GARCH(1,1) 300 day forecast sample.)

The size of the sample standard deviations, the large swings from maximum to minimum for some of the estimates, and the autocorrelation properties of the resulting standard deviation and correlation forecasts all indicate that one particular stationary GARCH(1,1) process did not generate the data through out the sample. Figure 2 plots the GARCH(1,1)-based forecasts over time. Among these forecasts, we see that variability is relatively high for the Yen volatility early in the period and the opposite is true for DM volatility. Furthermore, yen volatility exhibits a mild upward trend.

## **2.2 Option Implied Covariance Predictors**

Yen/mark options, which began trading in late 1991, have been the most actively traded PHLX cross-currency options. Although both British pound/mark and British

pound/yen options are also listed on the PHLX, these options trade much less actively. Among all PHLX currency options, the dollar/mark and dollar/yen are the most actively traded options.

Of the options in our study, the yen/mark option is traded less actively than either the dollar/mark or dollar/yen option. From Figure 1, we see that, relative to the dollar/mark and the dollar/yen options, the yen/mark options not only trade less actively, but also fail to trade on a significant number of trading days. Of 565 total trading days in our sample period, the yen/mark options trade on 340 days. For the non-traded days, we will simply use the last available trading day covariance matrix estimate. Clearly, this procedure should bias our results against finding significant forecasting information in the implied estimators.

Since the term of our value at risk measure is 30 or 60 days, we use only the two week - two month maturity subset of our full option sample. Two week - two month maturity yen/mark options trade on 261 days. For the days on which yen/mark options trade, we use all traded dollar/mark, dollar/yen and yen/mark options in our analysis. However, we exclude from the sample all options that violate early exercise conditions.<sup>10</sup>

To estimate the implied covariance matrix, we extend Whaley's (1982) univariate implied volatility estimation method to the bivariate domain. For the options we consider, three parameters must be estimated: two implied standard deviations and one implied correlation.

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<sup>10</sup> In the Bodurtha and Courtadon (1986) analysis of PHLX currency options, most exercise boundary-related pricing violations are explained by transaction costs.

Across the options, we identify the following system of equations:

$$\varepsilon_{t,DM}(\bullet) = C_{t,DM}(X, T, A) - \tilde{C}_t(S_{t,DM}, \sigma_{t,DM}, X, r_{t,US}(T), r_{t,DM}(T), T, A) \quad 1)$$

$$\varepsilon_{t,JY}(\bullet) = C_{t,JY}(X, T, A) - \tilde{C}_t(S_{t,JY}, \sigma_{t,JY}, X, r_{t,US}(T), r_{t,JY}(T), T, A) \quad 2)$$

$$\varepsilon_{t,DJ}(\bullet) = C_{t,DJ}(X, T, E) - \tilde{C}_t(S_{t,DJ}, \sigma_{t,DJ}, X, r_{t,JY}(T), r_{t,DM}(T), T, E) \quad 3)$$

$$\sigma_{t,DJ}^2 = \sigma_{t,DM}^2 + \sigma_{t,JY}^2 - 2\sigma_{t,DM,JY}, \quad \sigma_{t,DM,JY} = \rho_{t,DM,JY} \sigma_{t,DM} \sigma_{t,JY} \quad 4)$$

$C_{t,k}(\bullet)$  is the actual option price,  $k = \text{dollar/mark (DM), dollar/yen (JY),}$   
or yen/mark (DJ) on the option trade date,  $t$

$\tilde{C}_t(\bullet)$  is the model option price

$t$  is the option trade date

$X$  is the option exercise price

$T$  is the option maturity date

$A$  indicates a European or American option, which is valued by the  
currency option models of Garman-Kohlhagen (1983) or  
Bodurtha-Courtadon (1987) models, respectively<sup>11</sup>

$E$  indicates that only European options are traded on the yen/mark  
cross-currency options

$S_{t,k}$  is the time  $t$  currency  $k$  spot price

$r_{t,k}(T)$  is the time  $t$  currency  $k$  maturity  $T$  discount interest rate

$\sigma_{t,k}$  is the time  $t$  log spot price change standard deviation for currency  $k$

$\rho_{t,DM,JY}$  is the time  $t$  log spot price change correlation for the DM and JY.

In estimating the covariance matrix, we enforce its positive semi-definiteness by  
estimating the upper diagonal elements of the Cholesky decomposition of the matrix.

Therefore, the covariance matrix is decomposed as follows:

$$\Sigma = \begin{bmatrix} \sigma_{DM}^2 & \sigma_{DM,JY} \\ \sigma_{DM,JY} & \sigma_{JY}^2 \end{bmatrix} = C' C, \text{ and } C = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}.$$

<sup>11</sup> The respective models augment the standard Black-Scholes-Merton European option and Cox-Ross-Rubinstein American option pricing models with a continuous dividend adjustment, which is appropriate for currency options.

From Press, Teukolsky, Vetterling and Flannery (1992), a Levenberg-Marquardt algorithm is used to search over the parameter space. In the cases in which the covariance matrix is singular, the algorithm step size approaches infinity. Of the 261 days with yen/mark option trades, this condition occurs six times. As such estimates contradict our multivariate option pricing approach, we exclude the associated observations from our sample.<sup>12</sup>

To jointly estimate the implied covariance parameters, we minimize the sum of the squared pricing errors for a given day. Figure 4 plots the implied volatility and correlation estimates; Table 4 reports associated sample statistics. The implied volatility estimates fall between 10.5 and 12.5 percent, with 2.6 - 2.7 percent standard deviations. Dollar/mark and dollar/yen implied volatility estimates are skewed left; and correlation estimates are skewed right. Generally, the parameter estimates have relatively narrow tails. However, the mark/dollar implied volatility estimates are an exception to this pattern, with slightly fat tails (3.4 estimate vs. normal 3.0).

An important feature of the implied parameter time series is the autocorrelation structure of the estimates. The auto-correlation estimates are all on the order of 0.94 or greater. This finding is not surprising, because these estimates represent the option market's ex-ante volatility estimates averaged over the two week - two month option maturities in our sample. This phenomenon is similar to what we have also observed in the historical covariance matrix time series.

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<sup>12</sup> Nevertheless, including these observations yields qualitatively similar results. In these cases, we stop the algorithm just prior to forcing covariance matrix singularity. For our nonformal statistical analyses, including these degenerate cases is not problematic. However, if we want to formally test parameter estimate significance, then we must specifically treat the parameter estimates that approach (or lie on) the boundary of their sample space. A Monte Carlo or Bootstrap analysis must be used in this situation.

### 3. Predictability

Given information in historical and implied volatilities and correlations, we examine whether these statistics predict subsequent levels of volatility and correlation. To address this question, we conduct a simple regression-based test.<sup>13</sup>

Due to the high degree of autocorrelation in our independent and dependent variables, the prediction equation parameters are estimated by the Stock-Watson (1993) ordinary least squares procedure. The standard errors calculated in this procedure are of correct size in the presence of integrated regression variables. The associated t-statistics are also adjusted for autocorrelation and overlapping observations following Newey-West (1987). For the 30-day prediction horizon, the Newey-West lag adjustments are set at 40 (30 days overlap plus 10 days for potential autocorrelation). For the 60-day prediction horizon lag adjustments are set at 70.

Table 5 reports our 30-day forecast regression results. Among potential predictors, the lagged 30-day statistics, the weighted moving average statistics, and the implied standard deviations and correlation regressors have explanatory power. In all cases, the bivariate GARCH(1,1) regressor is dominated by another historical regressor. Furthermore, the direction of volatility forecast information in the GARCH estimator is of the wrong sign (negative.) Our implementation of the GARCH estimator does not perform well.

Forecasts of yen variability are especially problematic. The highest  $R^2$  is only 0.05. In this case, the simple 30-day lagged standard deviation regressor is combined with the implied volatility regressor. Nevertheless, the sign of the historical based estimators are all negative.

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<sup>13</sup> Following Lamourieux and Lastrapes (1993), an interesting extension of our work is Monte Carlo determination of the size and power of our test statistics. However, because of the American exercise feature of many of the options in our sample and the complexities of modeling multivariate stochastic covariance dynamics, we leave this work to future research.

The "best" yen variability forecast equation is the single variable implied volatility regression which only has a 0.003 adjusted  $R^2$ .

The mark volatility and mark/yen correlation equations provide much more appealing estimates and forecast performance. In these two cases, one of the two lagged squared error based forecast variables has most forecast information. The 30-day lagged standard deviation is the "best" predictor of future mark variability, while the weighted moving average (Riskmetrics) variable forecast equation provides the "best" correlation prediction. In both cases, the implied covariance measures provide incremental forecast information. These results indicate that a hybrid forecast equation which includes a fairly simple measure of lagged price variability and option implied covariance pricing is a reasonable alternative.

Caninna-Figlewski (1993) have found that volatility forecast performance is affected by the forecast horizon. To address this issue, we also analyze 60-day covariance forecasts. The forecast variables are leading 60-day standard deviations and correlation. Relative to the 30-day forecasts, two of the explanatory variables are the same and two differ. The (Riskmetrics) exponentially weighted moving average and implied covariance matrix estimates are the same. The simple lagged estimates and the GARCH estimates are adjusted for the 60 day forecast horizon.

Table 6 reports 60-day forecast horizon results. These results are, generally, similar to the 30-day forecast results. However, the few differences are of most interest. Specifically, the exponentially weighted moving average variables are relatively more significant. Of particular interest is the forecast performance of this variable for the mark/yen correlation. We view the associated 0.49  $R^2$  to be quite remarkable.

#### 4. Covariance Value at Risk

The high degree of autocorrelation in our implied parameter estimates suggests an analysis of the differenced covariance parameter time series. Additional motivation for analyzing changes in the implied parameter estimates comes from concern with the evaluation and management of option position value at risk. The variability of implied parameter changes can be used to generate confidence intervals for changes in option and other volatility-sensitive derivative portfolio values.

Figure 5 shows histograms of the changes in the implied parameter estimates. The changes are weighted by the square root of the time interval between the observations. In Table 7, we report associated sample statistics.<sup>14</sup> Among the implied volatility changes, the dollar/yen and dollar/mark volatility changes are similar, with a 1.4 standard deviation, right skewness, and moderately fat tails. The implied correlation changes have a 13.3 standard deviation estimate and are unskewed, with narrow tails.

Due to a combination of bid-ask bounce and other unspecified estimation errors, all of the implied parameter changes are strongly mean-reverting. Table 7 also reports first-order autocorrelation estimates on the order of negative 0.4. Therefore, we estimate the observation error as a first-order moving average process [MA(1)].

The MA(1)-adjusted estimates are reported in Table 8. The associated moving average parameter estimates are on the order of negative one-half. Table 8 also reports the

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<sup>14</sup> There appear to be two outlier observations in our sample. On February 25, 1993, a ten point drop occurred in dollar/yen volatility. This outlier can be smoothed by moving average adjustments for bid-ask bounce and volatility parameter estimation error. However, the ten point mark implied volatility drop on October 2nd, 1992 is a real phenomenon, which is also manifest in Knight-Ridder's CRB mark futures option implied volatility estimates.

autocorrelations with the MA(1) error component removed. From Table 8, we see that the residual implied parameter series is "whitened" reasonably well by this procedure.

Based on the MA(1) estimation error-corrected implied parameter series, we examine the correlation of the estimates over time. The dollar/mark and dollar/yen implied volatility changes are uncorrelated. All other correlations are positive.

Furthermore, and most importantly, high dollar/mark and dollar/yen implied volatilities are associated with higher implied correlations. This relationship suggests that as US dollar-based owners of mark and yen currency positions have faced higher levels of implied volatilities, the implicit measure of currency value at risk has increased (with increases in the implied correlation).

#### **4. Conclusion**

We present an initial jointly estimated set of implied volatility and correlation parameters for the mark and yen. Among these estimates, the new correlation parameter estimates illustrate dynamics and provide forecast explanatory power similar to that already documented for implied volatility estimates. The implied parameter estimates provide incremental explanatory power over historical-based estimates. Therefore, regulators, commercial and investment banks, corporations and institutional investors should consider both historical and implied volatility covariance parameter estimates for their "value at risk" measurement and management. As the PHLX currency options are the only active exchange-traded options which embody correlation pricing, our results point to expanded risk measurement information from trading of other "cross-" options.

Furthermore, we document a positive correlation between the mark and yen implied volatility estimates and the yen/mark implied correlation estimate. This ex-ante relationship is

similar to the ex-post historical relationship documented for international equity return time series by Longin and Solnik (1991) and King, Sentana and Wadhvani (1994). Therefore, both ex-ante and ex-post measures of time-varying covariances indicate lower cross-country diversification benefits and higher cross-country value at risk, in periods of high volatility. Importantly, our results indicate that this phenomenon is actually *priced* in the dollar/mark and dollar/yen foreign exchange markets.

Further modeling and empirical work should address the properties of individual implied parameter estimates as well as joint implied and time-series specifications of exchange rate changes and exchange rate covariance changes.<sup>15</sup> Analysis of the parameter estimates' sampling properties is an additional area of interest. Since certain overlapping horizon-based estimators manifest undesirable sampling properties, the sampling properties of joint dynamic covariance estimators require in-depth examination.<sup>16</sup>

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<sup>15</sup> Bates (1993) conducts an analysis of the dollar/mark options.

<sup>16</sup> For example, see Jegadeesh (1991) and Hodrick (1992).

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**Table 1: 30 day Forecast Standard Deviations and Correlation**

$$\text{Standard deviations - } \sigma_{t,k}^h = \sqrt{\sum_{j=1}^{i_t} \epsilon_{t+j,k}^2 / (i_t - 1)}, k = \text{DM or JY}$$

$$\text{Correlation - } \rho_{t,DM,JY}^h = \sum_{j=1}^{i_t} \epsilon_{t+j,DM} \epsilon_{t+j,JY} / (i_t - 1) / \left( \sigma_{t,DM}^h \sigma_{t,JY}^h \right)$$

$i_t$  is the number of trading days in the 30 days subsequent to date  $t$ .

	Standard			Maximum		Minimum		
	Average	Deviation	Skewness	Kurtosis	Value	Date	Value	Date
Volatility								
Dollar/yen	0.099	0.035	1.127	0.992	0.206	940211	0.040	921216
Dollar/mark	0.118	0.033	1.483	2.965	0.244	920910	0.063	931103
Correlation	0.531	0.182	-0.815	0.203	0.831	920821	-0.054	930713

	Autocorrelations								Augmented Dickey- Fuller
	1	2	3	4	5	6	7	8	
Volatility									
Dollar/yen	0.994	0.99	0.987	0.983	0.979	0.974	0.968	0.963	-3.963***
Dollar/mark	0.999	0.998	0.997	0.996	0.994	0.993	0.991	0.99	-2.784*
Correlation	0.995	0.99	0.984	0.98	0.976	0.973	0.97	0.967	-2.879**

\*, \*\*, \*\*\* are significant at the 10%, 5% and 1% levels, respectively

**Table 2: Weighted Moving Average Covariance Forecasts**

$$\sigma_{t,k}^w = (1-\lambda) \sum_{j=0}^{151} \lambda^j \varepsilon_{t-j+n,k}^2, \lambda = 0.97, k = \text{DM or JY}$$

$$\rho_{t,\text{DM},\text{JY}}^w = (1-\lambda) \sum_{j=0}^{151} \lambda^j \varepsilon_{t-j+n,\text{DM}} \varepsilon_{t-j+n,\text{JY}}, \lambda = 0.97$$

$$\varepsilon_{t,k} = \ln(S_t/S_{t-h}) - \mu_k h_t, h_t \text{ is the time between observations.}$$

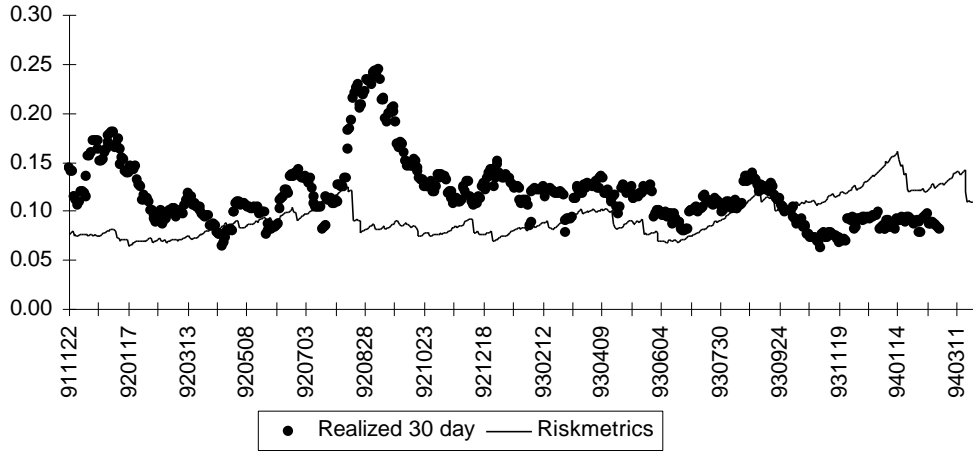
	Standard			Maximum		Minimum		
	Average	Deviation	Skewness	Kurtosis	Value	Date	Value	Date
Volatility								
Dollar/yen	0.093	0.019	0.957	0.496	0.161	940114	0.064	920117
Dollar/mark	0.128	0.022	0.839	0.254	0.201	930409	0.092	920917
Correlation	0.534	0.121	-1.001	0.602	0.720	911114	0.140	940214

	Autocorrelations								Dickey- Fuller
	1	2	3	4	5	6	7	8	
Volatility									
Dollar/yen	0.999	0.997	0.996	0.994	0.993	0.991	0.990	0.988	-2.090
Dollar/mark	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	-2.268
Correlation	1.000	0.999	0.999	0.999	0.998	0.998	0.998	0.998	-0.970

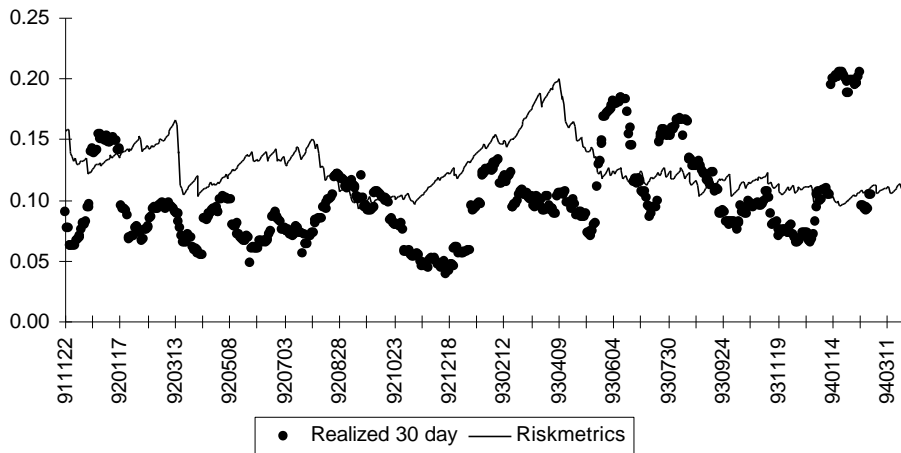
\*\* and \* are significant at the 1% and 5% levels, respectively

Figure 1

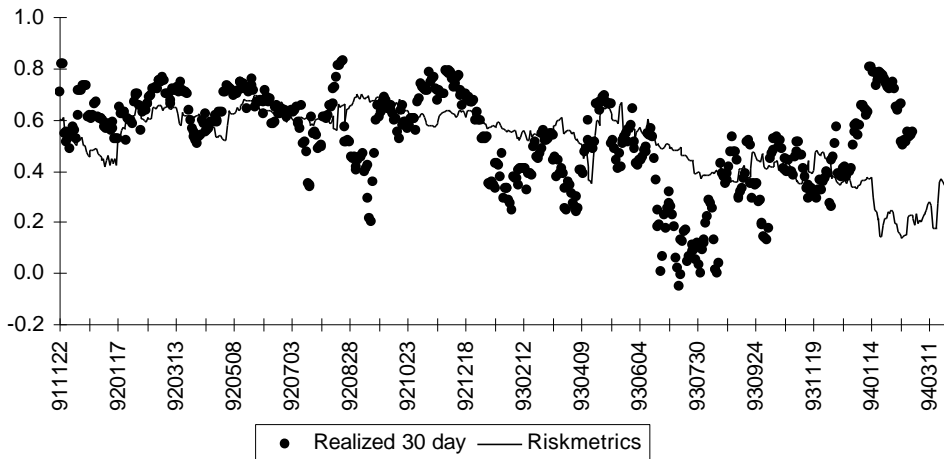
Monthly  $S_{JY}$



Monthly  $S_{DM}$



Monthly  $\Gamma_{DM,JY}$



**Table 3: GARCH(1,1) Covariance Forecasts**

Estimated Process 11/21/91-2/25/94

	$\mu_{DM}$	$\mu_{JY}$	$a_{11}$	$a_{12}$	$a_{22}$			
Coefficients	-0.0002	0.0003	-0.0055	0.0005	0.0051			
QMLE t-stats	0.740	1.152	3.331***	0.169	2.397***			
	$b_{11}$	$b_{12}$	$b_{21}$	$b_{22}$	$c_{11}$	$c_{12}$	$c_{21}$	$c_{22}$
Coefficients	0.4653	0.1782	0.2691	0.2999	0.5366	0.0227	0.4813	-0.0105
QMLE t-stats	6.486***	2.271**	3.768***	4.043***	0.826	0.032	0.644	0.011

300-day Rolling Forecasts

	$\mu_{DM}$	$\mu_{JY}$	$a_{11}$	$a_{12}$	$a_{22}$			
Average	-0.00006	0.0002	0.0029	0.0022	0.0040			
St. Deviation	0.00033	0.0003	0.0020	0.0018	0.0013			
Maximum	0.00078	0.0008	0.0067	0.0064	0.0075			
Minimum	-0.00076	-0.0004	-0.0041	-0.0030	-0.0026			
	$b_{11}$	$b_{12}$	$b_{21}$	$b_{22}$	$c_{11}$	$c_{12}$	$c_{21}$	$c_{22}$
Average	-0.397	0.151	-0.178	0.399	-0.313	-0.504	-0.111	-0.338
St. Deviation	0.287	0.258	0.117	0.159	0.235	0.359	0.315	0.430
Maximum	0.541	0.662	0.204	0.731	1.000	1.000	0.609	1.000
Minimum	-0.798	-0.423	-0.411	0.011	-1.000	-1.000	-1.000	-1.000

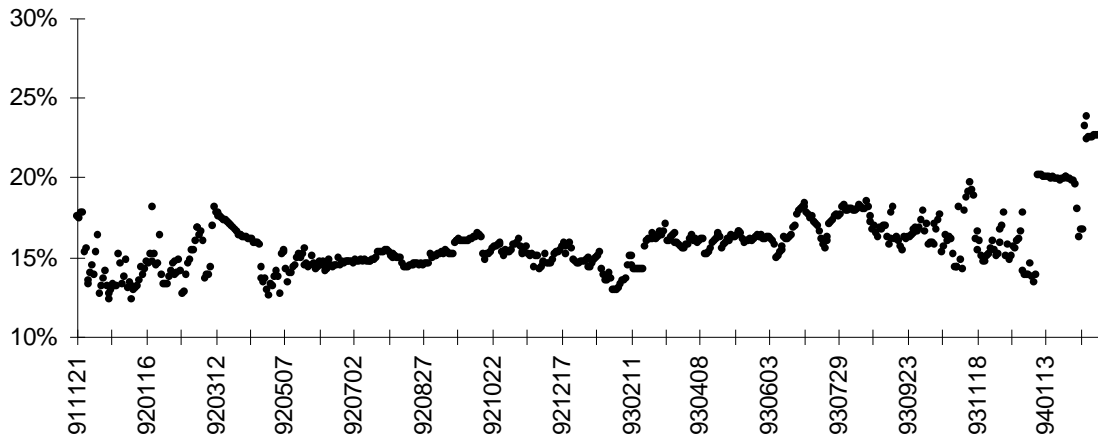
	Standard				Maximum Value	Maximum Date	Minimum Value	Minimum Date
	Average	Deviation	Skewness	Kurtosis				
Volatility								
Dollar/yen	0.175	0.034	2.186	8.631	0.398	930318	0.121	920312
Dollar/mark	0.229	0.029	1.033	4.582	0.435	920930	0.166	920220
Correlation	0.543	0.132	0.210	-0.542	0.957	920930	0.267	940222

	Autocorrelations								Dickey-Fuller
	1	2	3	4	5	6	7	8	
Volatility									
Dollar/yen	0.986	0.984	0.983	0.982	0.982	0.982	0.981	0.980	-1.919
Dollar/mark	0.994	0.994	0.994	0.994	0.994	0.994	0.994	0.995	-2.755*
Correlation	0.989	0.990	0.989	0.991	0.990	0.991	0.991	0.992	-1.120

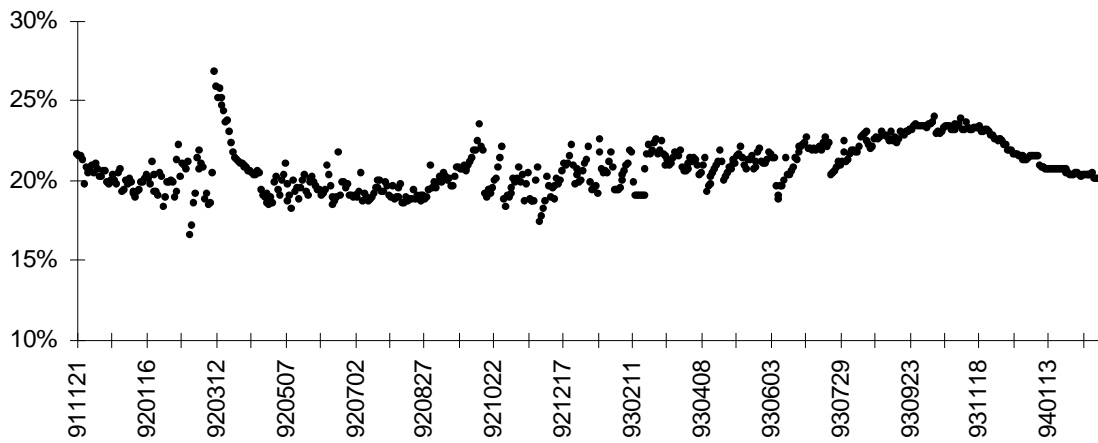
\*, \*\* and \*\*\* are significant at the 10%, 5% and 1% levels, respectively

Figure 2

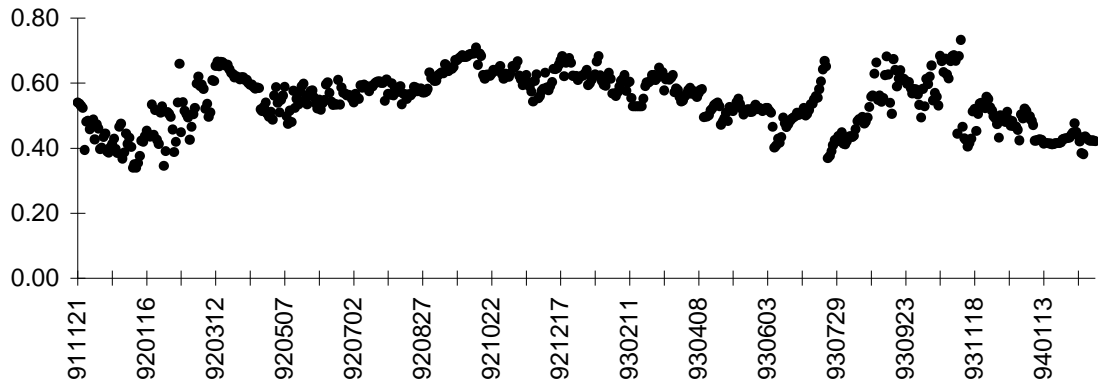
GARCH(1,1) JY<sub>S</sub>



GARCH(1,1) DM<sub>S</sub>

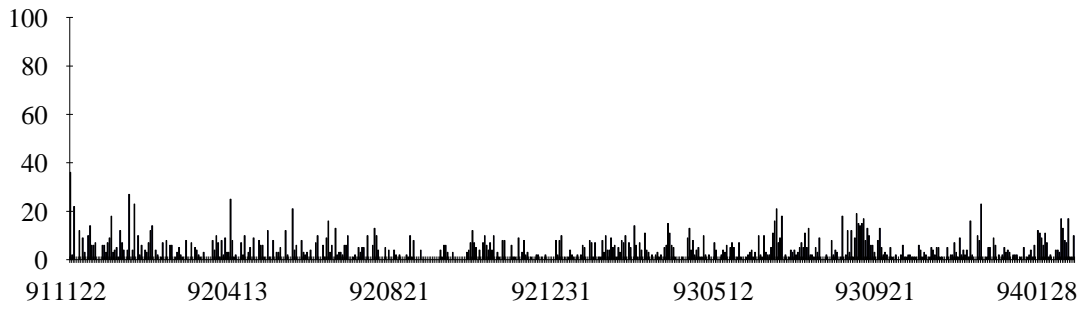


### GARCH(1,1) DMJY r

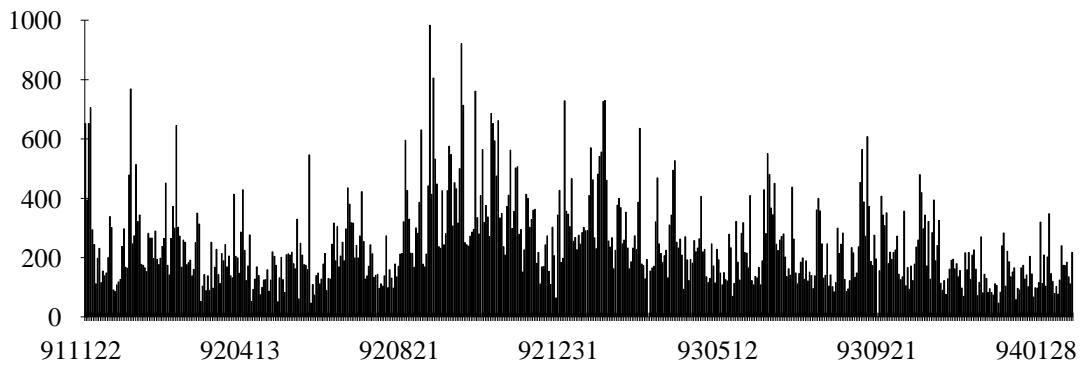


**Figure 3**

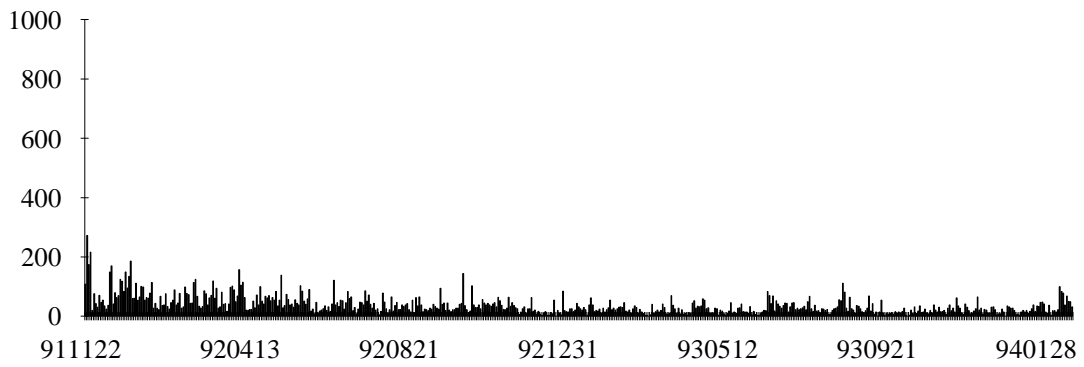
**DM-yen Option Trades**



**DM-dollar Options Trades**



**Yen-dollar Option Trades**



**Table 4: PHLX Mark and Yen Option Implied Parameters - Sample Statistics**

Implied parameters are estimated for Philadelphia Stock Exchange (PHLX) dollar/mark, dollar/yen and yen/mark currency options, which had between two weeks and two months time to expiration. The sample period is 11/22/91 - 2/24/94, and the observation frequency is daily. There were 540 trading days in this sample period, and our estimates are obtained for the 261 days on which the yen/mark options traded. All estimates are multiplied by 100.

	Standard				Maximum	Maximum	Minimum	Minimum
	Average	Deviation	Skewness	Kurtosis	Value	Date	Value	Date
Yen-dollar	0.106	0.025	0.733	-0.414	0.194	930701	0.067	920504
DM-dollar	0.125	0.029	1.545	3.41	0.253	921001	0.067	940127
Correlation	0.49	0.157	-0.009	1.141	1	911226	0.001	930329

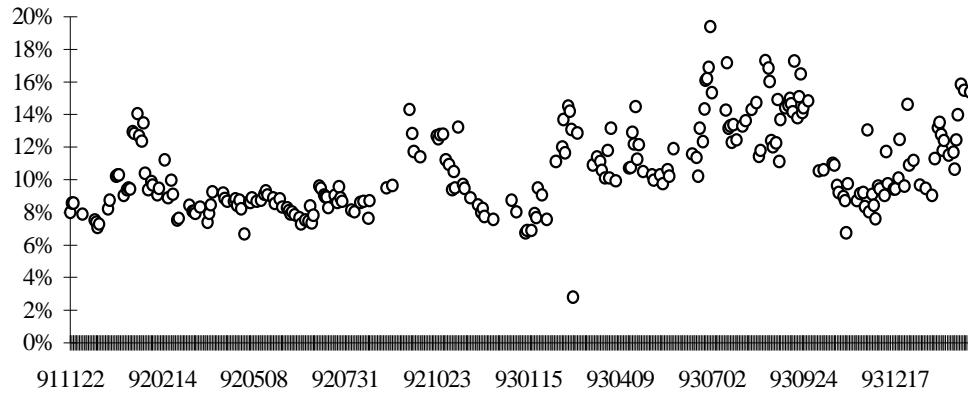
	Autocorrelations								Dickey- Fuller
	1	2	3	4	5	6	7	8	
Yen-dollar	0.992	0.99	0.988	0.986	0.985	0.983	0.982	0.981	-2.185
DM-dollar	0.996	0.993	0.991	0.989	0.987	0.987	0.987	0.986	-2.864*
Correlation	0.974	0.958	0.946	0.942	0.934	0.934	0.935	0.934	-4.814**

\*\* and \* are significant at the 1% and 5% levels, respectively

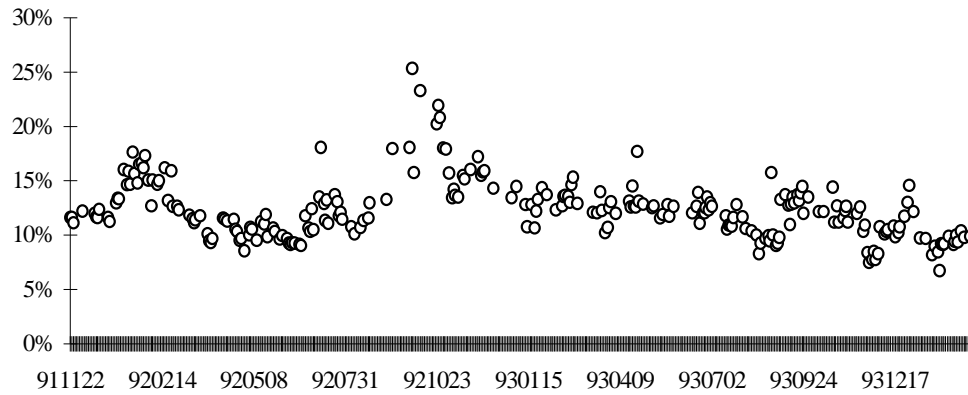


**Figure 4**

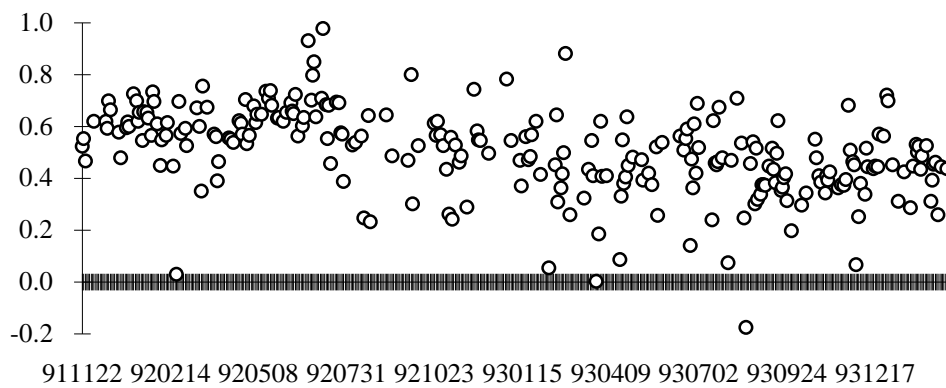
**Dollar-yen PHLX Option Implied Volatilities**



**Dollar-mark PHLX Option Implied Volatilities**



**Yen/mark PHLX Option Implied Correlations**



**Table 5: 30 Day Standard Deviation and Correlation Forecasts**

Standard Deviation

$$\sigma_{t,k}^h = \alpha_m + \beta_m \sigma_{t-30,k}^m + \delta_m \text{ISD}_{t-30} + \nu_{t,k}^m, k = \text{DM or JY}, m = \text{h,w.}$$

Correlation

$$\rho_{t,\text{DM},\text{JY}}^h = \alpha_m + \beta_m \rho_{t-30,\text{DM},\text{JY}}^m + \delta_m \text{ICOR}_{t-30} + \omega_{t,k}^m, m = \text{h,w.}$$

Forecast	$\alpha$	$\beta_h$	$\delta$	Adjusted R <sup>2</sup>	Dickey-Fuller	$\alpha$	$\beta_h$	$\delta$	Adjusted R <sup>2</sup>	Dickey-Fuller
$\sigma_{t,\text{JY}}^h$ implied	0.076 (4.11)		0.185 (1.08)	0.003	-3.556***					
Garch	0.098 (3.82)	-0.013 (0.09)		-0.002	-3.771***	0.094 (3.89)	-0.199 (1.10)	0.345 (1.52)	-0.006	-3.577***
Riskmetrics	0.107 (5.36)	-0.107 (0.52)		0.002	-3.802***	0.089 (3.90)	-0.194 (0.96)	0.238 (1.34)	0.007	-3.483***
Lagged	<b>0.101</b> (8.09)	<b>-0.046</b> (0.39)		<b>0.005</b>	<b>-3.918***</b>	<b>0.060</b> (3.43)	<b>-0.612</b> (2.82)	<b>0.917</b> (3.08)	<b>0.050</b>	-3.171**
$\sigma_{t,\text{DM}}^h$ implied	0.018 (1.04)		0.750 (5.62)	0.142	-3.206**					
Garch	0.255 (6.36)	-0.624 (3.59)		0.017	-2.274	0.134 (3.39)	-0.473 (3.21)	0.687 (5.36)	0.169	-3.413**
Riskmetrics	0.092 (3.60)	0.157 (0.78)		0.002	-1.919	-0.012 (0.46)	0.201 (1.24)	0.786 (6.04)	0.139	-3.450**
Lagged	<b>0.035</b> (2.72)	<b>0.658</b> (6.15)		<b>0.178</b>	<b>-3.484***</b>	<b>0.020</b> (1.22)	<b>0.331</b> (1.40)	<b>0.425</b> (1.50)	<b>0.183</b>	<b>-3.343**</b>
$\rho_{t,\text{DM},\text{JY}}^h$	0.147 (1.66)		0.720 (4.08)	0.008	-3.053**					
Garch	0.232 (2.62)	0.501 (3.11)		0.066	-3.073**	-0.058 (0.52)	0.448 (3.00)	0.650 (3.81)	0.084	-3.553**
Riskmetrics	<b>0.131</b> (1.70)	<b>0.714</b> (4.90)		<b>0.184</b>	<b>-3.181**</b>	<b>-0.038</b> (0.40)	<b>0.573</b> (3.89)	<b>0.495</b> (2.85)	<b>0.192</b>	<b>-3.434***</b>
Lagged	0.252 (4.13)	0.474 (4.27)		0.057	-3.380**	0.088 (1.08)	0.285 (2.28)	0.539 (2.87)	0.069	-3.616**

Stock-Watson (1993) and Newey-West (1987) adjusted t-stats are reported. See also Hamilton (1994) pp. 601-612. Standard OLS with White (1982) and Newey-West (1987) adjusted t-stats yield similar results. \*, \*\*, \*\*\* indicate significance at the 10%, 5% and 1% levels, respectively.

**Table 6: 60 Day Standard Deviation and Correlation Forecasts**

Standard Deviation  $\sigma_{t,k}^h = \alpha_m + \beta_m \sigma_{t-60,k}^m + \delta_m \text{ISD}_{t-60} + v_{t,k}^m, k = \text{DM or JY}, m = h, w.$

Correlation  $\rho_{t,DM,JY}^h = \alpha_m + \beta_m \rho_{t-60,DM,JY}^m + \delta_m \text{ICOR}_{t-60} + \omega_{t,k}^m, m = h, w.$

Forecast	$\alpha$	$\beta_h$	$\delta$	Adjusted $R^2$	Dickey-Fuller	$\alpha$	$\beta_h$	$\delta$	Adjusted $R^2$	Dickey-Fuller
$\sigma_{t,JY}^h$ implied	0.106 (5.62)		-0.137 (0.79)	0.005	-2.475					
Garch	0.157 (6.54)	-0.370 (2.76)		0.041	-2.655*	0.154 (6.44)	-0.541 (3.04)	0.320 (1.43)	0.011	-2.741*
Riskmetrics	<b>0.150</b> <b>(8.56)</b>	<b>-0.626</b> <b>(3.45)</b>		<b>0.094</b>	<b>-3.000**</b>	<b>0.149</b> <b>(6.96)</b>	<b>-0.634</b> <b>(3.34)</b>	<b>0.017</b> <b>(0.10)</b>	<b>0.093</b>	<b>-3.061**</b>
Lagged	0.114 (7.12)	-0.221 (1.43)		0.024	-2.297	0.099 (4.99)	-0.507 (1.75)	0.412 (1.21)	0.011	-2.073
$\sigma_{t,DM}^h$ implied	0.010 (0.48)		0.780 (4.98)	0.140	-2.206					
Garch	0.240 (5.26)	-0.578 (2.92)		0.022	-1.540	0.115 (2.42)	-0.433 (2.45)	0.732 (4.77)	0.162	-2.358
Riskmetrics	0.042 (1.50)	0.524 (2.41)		0.052	-1.143	<b>-0.067</b> <b>(2.22)</b>	<b>0.564</b> <b>(3.08)</b>	<b>0.823</b> <b>(5.59)</b>	<b>0.191</b>	-2.090
Lagged	<b>0.026</b> <b>(1.48)</b>	<b>0.692</b> <b>(4.83)</b>		<b>0.178</b>	<b>-2.814</b>	0.018 (0.87)	0.512 (1.46)	0.234 (0.60)	0.185	-2.672*
$\rho_{t,DM,JY}^h$	0.085 (0.94)		0.761 (4.20)	-0.027	-1.902					
Garch	0.082 (0.94)	0.703 (4.43)		0.133	-2.423	-0.220 (2.02)	0.646 (4.42)	0.678 (4.07)	0.118	-2.626*
Riskmetrics	-0.142 (2.34)	1.164 (10.21)		0.495	-3.138**	<b>-0.257</b> <b>(3.42)</b>	<b>1.068</b> <b>(9.13)</b>	<b>0.336</b> <b>(2.44)</b>	<b>0.493</b>	-3.045**
Lagged	0.259 (3.43)	0.385 (2.73)		0.055	-1.929	0.075 (0.82)	0.043 (0.25)	0.737 (3.17)	-0.018	-1.915

Stock-Watson (1993) and Newey-West (1987) adjusted t-stats are reported. See also Hamilton (1994) pp. 601-612. Standard OLS with White (1982) and Newey-West (1987) adjusted t-stats yield similar results. \*, \*\*, \*\*\* indicate significance at the 10%, 5% and 1% levels, respectively.

**Table 7: PHLX DM and Yen Option Implied Parameter Changes - Sample Statistics**

(The histograms associated with these estimates are plotted in Figure 5.)

Implied parameters are estimated for Philadelphia Stock Exchange (PHLX) dollar/mark, dollar/yen and yen/mark currency options, which had between two weeks and two months time to expiration. The sample period is 11/22/91 - 2/24/94, and the observation frequency is daily. There were 540 trading days in this sample period, and our estimates are obtained for the 261 days on which the yen/mark options traded. All estimates are multiplied by 100.

	Average	Standard Deviation	Skewness	Kurtosis	Maximum Value	Maximum Date	Minimum Value	Minimum Date
Volatility								
Dollar/yen	-0.011	1.353	-1.691	14.28	4.502	930302	-10.27	930225
Dollar/mark	-0.033	1.388	-0.963	11.847	6.317	930826	-9.542	921002
Correlation	-0.109	13.338	-0.006	2.036	43.282	930331	-49.916	921002

**Implied Parameter Changes - Autocorrelations**

	1	2	3	4	5	6	7	8
Volatility								
Dollar/yen	-0.381	0.089	-0.061	0.033	-0.072	0.046	-0.14	0.182
Dollar/mark	-0.410	0.029	0.04	-0.080	0.078	-0.007	-0.018	-0.033
Correlation	-0.401	-0.093	0.06	-0.052	0.150	-0.114	0.008	0.046

Table 8

Given the high first order autocorrelation of changes in the implied parameter estimates that is reported in Table 1, a first order moving average process is fitted to the change estimates. The sample statistics for the associated filtered process are then reported. The sample period is 11/22/91 - 2/24/94, and the observation frequency is daily. There were 540 trading days in this sample period, and our estimates are obtained for the 261 days on which the yen/mark options traded.

**Changes in Implied Parameters - MA(1) process Estimates**

	$\theta$	QMLE-t	Standard Deviation
Volatility			
Dollar/yen	-0.396	6.6	1.25
Dollar/mark	-0.464	4.16	1.25
Correlation	-0.589	12.22	11.55

**Changes in Implied Parameters - MA(1)-adjusted Autocorrelations**

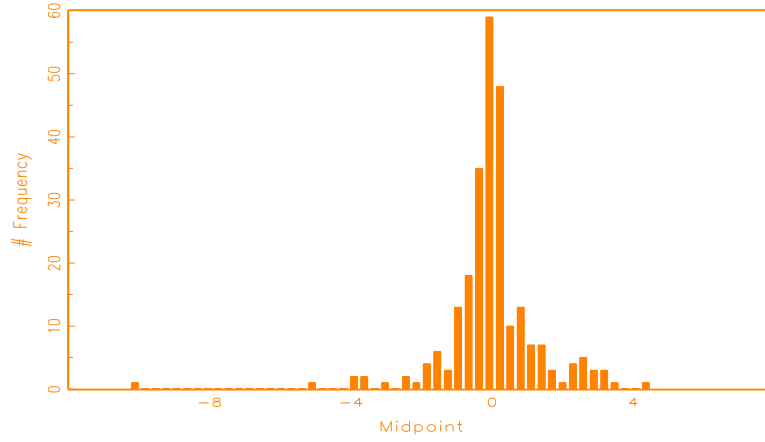
	1	2	3	4	5	6	7	8
Volatility								
Dollar/yen	-0.018	0.07	-0.04	-0.008	-0.082	-0.018	-0.105	0.125
Dollar/mark	-0.022	0.034	0.037	-0.043	0.064	0.003	-0.035	-0.046
Correlation	0.012	-0.071	0.04	0.025	0.134	-0.057	-0.006	0.025

**MA(1) Filtered Changes in Implied Parameters - Correlations**

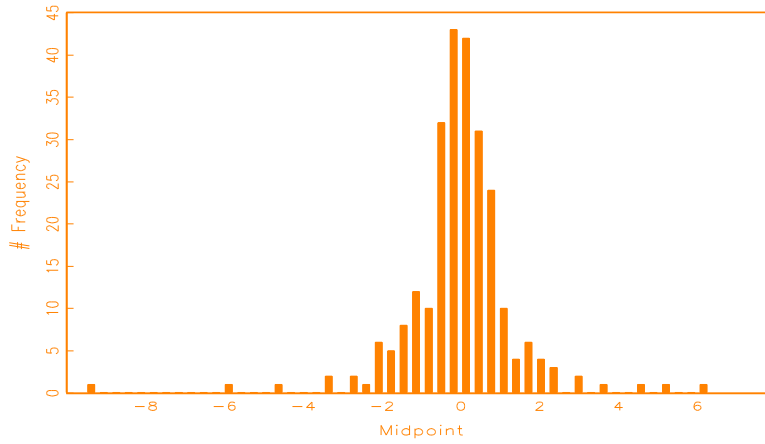
	Volatility Dollar/yen	Volatility Dollar/mark	Correlation
Volatility			
Dollar/yen	1.000		
Dollar/mark	-0.010	1.000	
Correlation	<b>0.116</b>	<b>0.430</b>	1.000

**Figure 5**

**Changes in Dollar/yen Implied Volatility**



**Changes in DM-dollar Implied Volatility**



**Changes in Yen/mark Implied Correlation**

