

SFAS 133 and IAS-39 Hedge Effectiveness and Objective Probability Derivative Valuation

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From a pedagogic perspective, the necessary relationships between the true/real world and derivative/fair-value world are often not treated in respective valuation and derivative/option paradigms. Hence, important misconceptions and inconsistencies may enter financial management, financial accounting and related regulation. From the applied perspective, heterogeneous real world probabilities and risk-neutral valuation probabilities can and do lead to misconceptions. This work addresses potential misconceptions in two ways. First, we provide a simple binomial process-based valuation treatment that is consistent in both the real and fair value worlds. Second, we highlight the importance of considering both worlds in the context of the most fundamental of derivative related applications, determining hedge effectiveness.

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From a pedagogic perspective, the necessary relationships between the true/real world and derivative/fair-value world are often not treated in respective valuation and derivative/option paradigms. Hence, important misconceptions and inconsistencies may enter financial management, financial accounting and related regulation. From the applied perspective, heterogeneous real world probabilities and risk-neutral valuation probabilities can and do lead to misconceptions. This work addresses potential misconceptions in two ways. First, we provide a simple binomial process-based valuation treatment that is consistent in both the real and fair value worlds. Second, we highlight the importance of considering both worlds in the context of the most fundamental of derivative related applications, determining hedge effectiveness.

SFAS 133 and IAS-39 hedge effectiveness measures are based on “true” or objective probability measures, and not “risk-neutral” probabilities.¹ For SFAS 133 net purchased option cash flow hedges, the Derivatives Implementation Group released a rule interpretation (G20) that permits certain cash flow hedge time value changes to be credited or debited to other comprehensive income and not to earnings.²

As originally promulgated the rule was harshly criticized by hedging entities with extensive long-term forecasted exposures.³ In regard to this interpretation, we make two points. First, certain financial engineering structures achieve the same objective, Bodurtha-Thornton (2002). Second, the hedge effectiveness component of the interpretation requires matching option fair value changes with option expected value changes. Therefore, the directive mixes the risk-neutral and true or objective probability measures.

¹ Louis (2000) and Shell (2000) are examples.

² A previous version of our work argued in the spirit of this directive, “The FAS 133 Cash Flow Hedge: Financial-Engineering, Finance, and Financial-Accounting Perspectives,” January 2001.

³ This issue is of paramount concern for firms that hedge longer-term forecast transactions. MacKay-Niedzielski (2000). It is interesting to us that the oscillating earnings phenomenon that has been depicted only arises for option cash flow hedges of three quarters or more. For shorter terms and for options that are initiated near-the-money, earnings adjustments will mostly be negative and the FAS 133 rule only reallocates these costs across time and state.

Since these measures are not equal, hedge effectiveness is imperfect if management designates the option, as opposed to merely its intrinsic value, as the hedging instrument. Moreover, hedge effectiveness will erode for longer maturity and systematically risky underlying risks.

1) Risk-Neutral or Martingale Probabilities and True or Objective Probabilities

We link the hedge effectiveness and option treatments of SFAS 133 and IAS 39 by linking the objective and risk-neutral probability measures and associated values. To simply define this link, we must only specify an instantaneous risk-premium for the underlying exposure.

An estimate of this risk premium, α , is implicit in expected price change and expected return calculations, which are necessary to evaluate hedge effectiveness. Specifically, the annualized expected logarithmic change in the underlying price, μ , is equal to the following:

$$\mu = \alpha + r - \frac{\sigma^2}{2} \quad 1)$$

Since SFAS 133 hedge effectiveness assessment requires a volatility, σ , we can calculate a risk premium, α :

$$\alpha = \mu + \frac{\sigma^2}{2} - r \quad 2)$$

An underlying may not manifest a risk premium under one of two conditions: the risk in the underlying is diversifiable or marginal investors are risk-neutral. In either case, we would have $r = \mu + \frac{\sigma^2}{2}$. The variance adjustment to any expected return is

necessary to account for the non-linearity or convexity in log-normally distributed prices. A 50/50 bet on a log-normal price outcome is not a fair bet.⁴

Given a risk premium estimate and our assumed binomial underlying price process, the true (objective) probability associated with a risk-neutral probability (equation 1) may be defined as follows:⁵

$$q = \frac{e^{(\alpha+r-y)h} - d}{u - d} \quad 3)$$

Standard discounted expected cash flow valuation of the underlying itself is appropriate for any maturity when we use the risk-adjusted discount rate, equation 1), and the true probabilities, q. To illustrate this correspondence, we return to our example context.

In Figure 1, the example spot price outcomes under both the risk-neutral and true probability measures are depicted. These measures are “equivalent” in the sense that the associated price outcome sets are the same. The risk premium on the underlying is

⁴ For example, assume that the underlying value goes up or down with equal probability at a 10% continuously compounded rate. Unlike an equally weighted bet with 10% gain or loss calculated on a simple interest basis, the fair value of this continuously compounded return bet is greater than one dollar. $0.5e^{0.1} + 0.5e^{-0.1} = 1.005 \neq 1$. The variance of this bet is about 1%, $0.5(e^{0.1} - 1.005) + 0.5(e^{-0.1} - 1.005) = 0.01$. In the risk-neutral case, any \$1 bet that pays off immediately must have no expected gain. If we subtract one-half of the variance from any continuously compounded risky bet's pay out, then we satisfy this no arbitrage condition: $0.5e^{(0.1-0.01/2)} + 0.5e^{(-0.1-0.01/2)} = 1$. The analogous $\sigma^2/2$ adjustment is required in equations a-1), a-2) and through out this appendix.

⁵ Cox-Ross-Rubinstein (1976) and Rubinstein (1976) implicitly use this specification, which follows from the following stochastic differential equation: $\frac{dS}{S} = \left(\mu + \frac{\sigma^2}{2} \right) dt + \sigma dz$. For this case, $\left[\frac{\sum_{i=1}^n \left(\ln \left(S_{t+ih} / S_{t+(i-1)h} \right) \right) / n}{h} \right] / h = \hat{\mu}$ and $\left[\frac{\sum_{i=1}^n \left(\ln \left(S_{t+ih} / S_{t+(i-1)h} \right) - \hat{\mu}h \right)^2 / (n-1)}{h} \right] / h = \hat{\sigma}^2$. $\hat{\alpha} = \hat{\mu} + \frac{\hat{\sigma}^2}{2} - r$ is an unbiased risk premium estimate.

assumed to be 4%. From equation 1), the true probability expected value discount rate is 9%, or 4% in excess of the 5% riskless rate.⁶

In each period of time length, h , the risk-neutral probability, p , is 48.23%, and the true probability, q , is 51.77%. In Table 1 below, we show the values and associated probabilities of the underlying price outcomes for both probability alternatives over 0.75 years:

Table 1		Risk neutral probability of outcome	True probability of outcome
Spot outcomes	# of paths to outcome		
152.8	1	1.26%	1.93%
132.7	2	8.11%	10.76%
115.2	15	21.76%	25.07%
100.0	20	31.13%	31.13%
86.8	15	25.06%	21.75%
75.4	6	10.76%	8.10%
65.4	1	1.92%	1.26%
Expected Value		100	103.05
Add 5% Yield		103.05	106.98
Discounted value @ Risk neutral rate =5%		100	N/A
Discounted value @ Risk-adjusted rate = 9%		N/A	100

The underlying may be equivalently valued under the risk-neutral or true probability measures.

Given the underlying price process, our example derivative values are calculated under the risk-neutral probability measure. We now illustrate derivative valuation equivalence under both the risk-neutral and true probability measures. However, it will become clear that derivatives valuation under the true measure is much more complicated

⁶ This discount rate is consistent with our assumed 20% volatility estimate and a 7% annualized expected logarithmic price change. The difference between the expected logarithmic price change and the expected value discount rate is equal to the variance divided by two. This “convexity” adjustment is necessary for any multiplicative underlying price process. Alternatively, this adjustment may also be assigned to the risk-neutral probability or the fundamental up and down parameter definitions of the binomial model. We have chosen a particular parameterization for ease of exposition. See Jarrow-Rudd (1983), Cox-Rubinstein (1985), Bodurtha-Courtadon (1987), Jennergren-Naslund (1993) and Hull (2000) for more discussion.

than under the risk-neutral measure. Under the true measure, the risk and expected return of the derivative change at every time and with each price change.

In fact, derivative valuation under the true measure requires one of two assumptions: 1) An estimate of the underlying risk premium, which formally requires that the marginal utility of the representative investor for the underlying of concern is known for all times and possible prices.⁷ 2) Option-delta hedging is possible, so that the option price elasticity with respect to the underlying price may be recovered at each time and price outcome. Since our risk-neutral probability-based option valuation requires the second assumption, we construct the true measure analogue under this condition.

Figure 2 depicts the example call option prices, and Figure 3 states the associated delta position in the underlying. Figure 3 records evolution of the option delta as time and spot price evolve.

A long delta fraction underlying position that is funded by time and state-dependent borrowing will provide a levered underlying position that is equivalent to the associated call across time and spot price outcomes.⁸

$$\Delta u^n d^m = e^{-yh} \frac{Cu^{n+1}d^m - Cu^n d^{m+1}}{Su^{n+1}d^m - Su^n d^{m+1}} \quad 4)$$

The option delta provides one component of a necessary adjustment to option value discount rates under the true probability measure, q . The full adjustment is based

⁷ Part of Rubinstein's (1995) analysis of employee stock options (FAS 123) is done in this manner. Constantinides (1978) and McDonald-Siegel (1985) provide treatments in the Capital Asset Pricing Model – CAPM context. Note their stochastic differential equation is $\frac{dS}{S} = \mu' dt + \sigma dz$, $\mu' = \mu - \frac{\sigma^2}{2}$. Their approaches may be applied in “non-marketable” underlying cases.

⁸ The state specific borrowing is defined $Bu^n d^m = e^{-th} (u Cu^n d^{m+1} - d Cu^{n+1} d^m) / (u-d)$.

on the option elasticity, and this elasticity is the delta times the ratio of underlying spot price and the option price.

$$\varepsilon u^n d^m = \Delta u^n d^m \frac{S u^n d^m}{C u^n d^m} \quad 5)$$

Figure 4 provides the example binomial call option elasticity process. These values are calculated from the corresponding underlying spot prices, call option prices and call option deltas in Figures 1, 2 and 3, respectively. The #N/A or “not applicable” elasticities are those for which the option is certain to finish out-of-the-money, and the elasticity denominator is zero.

Cox-Rubinstein (1985, pp. 185-196) have shown that option discount rates are equal to the underlying discount rate times the time- and underlying price-dependent option elasticity. Specifically,

$$r u^n d^m = \ln \left(\varepsilon u^n d^m \left(e^{(\alpha+r)h} - e^{rh} \right) + e^{rh} \right) / h \quad 6)$$

The associated time- and underlying spot price-dependent call option discount rates are depicted in Figure 5. Among the rates, the lowest discount rates are associated with the most in-the-money options. The option discount rates are bounded below by the underlying discount rate, 9%. As in the case of the corresponding option elasticities, the #N/A or “not applicable” rates are those for which the option is certain to finish out-of-the-money.

Based on the option risk-adjusted discount rates, we may value the option cash flows by recursion from maturity backward to time zero. The recursion used to calculate discounted expected values is the following:

$$C u^n d^m = e^{-r u^n d^m h} \left(q C u^{n+1} d^m + (1-q) C u^n d^{m+1} \right) \quad 7)$$

Figure 6, presents the time- and underlying price-dependent option value and the associated state-specific objective probabilities. As must be the case, the option price outcomes are equivalent to the outcomes in the risk-neutral case (Figure 2.) This direct subordination of option price outcomes to the common underlying price set under both true and risk-neutral measures is a necessary condition for market clearing.

2) Longer-term Relationships and DIG G20 – Cash Flow Hedge “Effectiveness”

Hedge effectiveness and fair value option treatments of SFAS 133 (and IAS-39) are linked by the objective and risk-neutral probability measures and associated values. Our extended example shows that one may equivalently value derivatives under the risk-neutral and true probability measures. This exercise required short-term expectations calculation and associated discounting. Based on this short-term analysis, a longer-term alternative may also be specified.

We may define the expected value for the option under the binomial distribution as follows:

$$E(C_t) = S e^{(\alpha+r-y)t} B[a, n, q'] - X B[a, n, q] \quad 8)$$

$B[]$ is the complementary binomial distribution; “ a stands for the minimum number of upward moves the stock must make over the next n periods for the call to finish in-the-money (a will be the smallest nonnegative integer such that $u^a d^{n-a} S > X$),”⁹ and

$$q = \frac{e^{(\alpha+r-y)h} - d}{u - d}, \quad q' = u e^{-(\alpha+r-y)h} q. \quad 10)$$

⁹ The corresponding risk-neutral expectation is defined in Cox-Ross (1985, pg. 177.)

$$\tilde{E}(C_t) = S e^{(r-y)t} B[a, n, p'] - X B[a, n, p], \quad p = \frac{e^{(r-y)h} - d}{u - d}, \quad p' = u e^{-(r-y)h} p.$$

¹⁰ In our example the term risk-adjusted discount rate for the option expected value is 35.9%, $\mu_{ct} = \ln(E(C_t)/C_0)/t$. This risk-adjusted discount rate is not a simple transformation of the period-

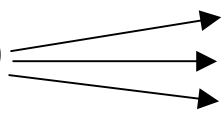
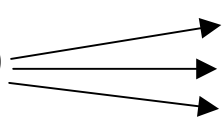
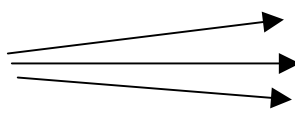
Following Rubinstein (1976) and O'Brien (1986), we allow the number of periods, n , to go to infinity (h to go to zero). We have

$$E(C_t) = Se^{(\alpha+r-y)t} N(z + \sigma\sqrt{t}) - XN(z), \quad 9)$$

where N is the standard normal distribution, and $z = \frac{\ln(S/X) + (\alpha + r - \sigma^2/2 - y)t}{\sigma\sqrt{t}}$

The following Figure depicts the example call option fair value and expected value evolution. Over one quarter, option fair value and expected value follow one of three paths:

Changes in Call Option Fair Value and Expected Value

	0	0.25	
Expected Values			
8.345 = EC0		18.23 = ECu2 6.42 = ECud 1.09 = ECd2	9.89 = ECu2 - EC0 -1.93 = ECud - EC0 -7.25 = ECd2 - EC0
Fair Values			
6.377 = C0		15.74 = Cu2 5.17 = Cud 0.80 = Cd2	9.36 = Cu2 - C0 -1.21 = Cud - C0 -5.58 = Cd2 - C0
Hedge Effectiveness = $\frac{\text{Expected Value Change}}{\text{Fair Value Change}}$		105.6% in u2 159.5% in ud 130.1% in d2	

by-period discount rates which are listed in Figure A-3. As noted in the original Rubinstein (1976, pg 417) work, the usefulness of the term expected value risk-adjusted discounting method is "hampered primarily by measurement of this rate."

Clearly the expected values and the expected value changes differ from the option fair values and the fair value changes. Option expected values and fair values do not evolve equivalently.

In fact, the values diverge significantly enough that DIG G-20 hedge effectiveness rule will not be satisfied. We indicate this phenomenon in the context of the first quarter changes. In our original example, prices change at each half quarter so that the first quarter changes are the compounded changes over two periods.

After a quarter, if the underlying either return to its initial value or fall in value, then the “80-125” hedge effectiveness rule would be violated. In returning to the initial value, an up or down price move is followed by the respective down or up move to outcome ud . At this point, the expected cash flow change is -1.93 and the time value change is only -1.21 . Hence, the cash flow hedge effectiveness test ratio is $159.5\% = -1.93/-1.21$. Analogously, for two down price changes, the effectiveness test ratio is $130.1\% = -7.25/-5.53$. Clearly, the DIG G-20 hedge effectiveness exception that permits reporting option cash flow hedge time value changes in other comprehensive income does not apply in this example.

The relatively large example hedge ineffectiveness will be manifest generally. Therefore, the Bodurtha-Thornton (2002) time-value swap structure that permits amortization of an option premium may be useful for cash flow hedges that don't qualify for DIG G-20 treatment. Fundamentally, equating risk-neutral probability based fair values with expected option values is inappropriate.

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Figure 2

Call Option Value Process Evolution

0	0.125	0.25	0.375	0.5	0.625	0.75
						$Cu^6 = 52.85 = \text{Max}(Su^6 - X, 0)$
					$Cu^5 = 42.15$	$Cu^5_d = 32.69 = \text{Max}(Su^5_d - X, 0)$
			$Cu^4 = 32.28$		$Cu^4_d = 23.48$	$Cu^4_{d^2} = 15.19 = \text{Max}(Su^4_{d^2} - X, 0)$
		$Cu^3 = 23.19$		$Cu^3_d = 15.00$	$Cu^3_{d^2} = 7.28$	$Cu^3_{d^3} = 0.00 = \text{Max}(Su^3_{d^3} - X, 0)$
	$Cu = 10.20$	$Cu^2 = 15.74$		$Cu^2_d = 8.99$	$Cu^2_{d^2} = 3.49$	$Cu^2_{d^4} = 0.00 = \text{Max}(Su^2_{d^4} - X, 0)$
$C_0 = 6.377$		$Cu_d = 5.17$		$Cu_d^2 = 1.67$	$Cu_d^3 = 0.00$	$Cu_d^5 = 0.00 = \text{Max}(Su_d^5 - X, 0)$
	$Cd = 2.89$	$Cd^2 = 0.80$		$Cd^2 = 0.00$	$Cd^4 = 0.00$	$Cd^6 = 0.00 = \text{Max}(Sd^6 - X, 0)$
			$Cd^3 = 0.00$		$Cd^5 = 0.00$	
				$Cd^4 = 0.00$		

Figure 3

Delta Hedge Process Evolution

	0	0.125	0.25	0.375	0.5	0.625
$\Delta u^n d^m = \exp(-y \cdot h) \cdot (Cu^{n+1}d^m - Cu^n d^{m+1}) / (Su^{n+1}d^m - Su^n d^{m+1})$						$\Delta u_5 = 0.994$
				$\Delta u_3 = 0.981$	$\Delta u_4 = 0.988$	$\Delta u_4 d = 0.994$
		$\Delta u_2 = 0.866$		$\Delta u_3 d = 0.988$		$\Delta u_3 d_2 = 0.994$
$\Delta 0 = 0.513$	$\Delta u = 0.692$		$\Delta u_2 d = 0.753$		$\Delta u_2 d_2 = 0.511$	
	$\Delta d = 0.329$	$\Delta u d = 0.513$		$\Delta u_2 d_2 = 0.511$		$\Delta u_2 d_3 = 0.000$
		$\Delta d_2 = 0.135$	$\Delta u d_2 = 0.263$		$\Delta u d_3 = 0.000$	$\Delta u d_4 = 0.000$
			$\Delta d_3 = 0.000$		$\Delta u d_4 = 0.000$	
				$\Delta d_4 = 0.000$		$\Delta d_5 = 0.000$

Figure 4

Option Elasticity Process Evolution

	0	0.125	0.25	0.375	0.5	0.625
$\epsilon u^n d^m = \Delta u^n d^m \cdot S u^n d^m / C u^n d^m$						$\epsilon u^5 = 3.358$
					$\epsilon u^4 = 4.059$	
				$\epsilon u^3 = 5.232$		$\epsilon u^4 d = 5.232$
		$\epsilon u^2 = 6.337$		$\epsilon u^2 d = 8.994$	$\epsilon u^3 d = 7.583$	
$\epsilon_0 = 8.052$	$\epsilon u = 7.275$		$\epsilon u d = 9.936$		$\epsilon u^2 d^2 = 14.648$	$\epsilon u^3 d^2 = 14.648$
	$\epsilon d = 10.609$		$\epsilon d^2 = 14.648$	$\epsilon u d^2 = 14.648$		#N/A
				#N/A	#N/A	#N/A
					#N/A	#N/A
						#N/A

Figure 4

Option Elasticity Process Evolution

	0	0.125	0.25	0.375	0.5	0.625
$\epsilon u^n d^m = \Delta u^n d^m \cdot S u^n d^m / C u^n d^m$						$\epsilon u^5 = 3.358$
					$\epsilon u^4 = 4.059$	
				$\epsilon u^3 = 5.232$		$\epsilon u^4 d = 5.232$
		$\epsilon u^2 = 6.337$		$\epsilon u^2 d = 8.994$	$\epsilon u^3 d = 7.583$	
$\epsilon_0 = 8.052$	$\epsilon u = 7.275$		$\epsilon u d = 9.936$		$\epsilon u^2 d^2 = 14.648$	$\epsilon u^3 d^2 = 14.648$
	$\epsilon d = 10.609$		$\epsilon d^2 = 14.648$	$\epsilon u d^2 = 14.648$		#N/A
				#N/A	#N/A	#N/A
					#N/A	#N/A
						#N/A

Figure 5

Risk-adjusted Discount Rate Process Evolution under the True-Objective-Q Measure

Underlying Risk premium= $\alpha=4.00\%$
 Risk-adjusted underlying discount rate= $r = \alpha + r = 9.00\%$

$$r_u^n d^m = \ln(\varepsilon u^n d^m (e^{er^*h} - e^{r^*h}) + e^{r^*h})/h$$

	0	0.125	0.25	0.375	0.5	0.625
						$ru^5=18.35\%$
					$ru^4=21.11\%$	
				$ru^3=25.71\%$		$ru^4d=25.71\%$
		$ru^2=30.02\%$			$ru^3d=34.84\%$	
	$ru=33.65\%$		$ru^2d=40.28\%$			$ru^3d^2=61.68\%$
		$rud=43.88\%$		$ru^2d^2=61.68\%$		
$r_0=36.65\%$		$rd=46.45\%$	$rud^2=61.68\%$			$\#N/A$
		$rd^2=61.68\%$			$\#N/A$	
			$\#N/A$		$\#N/A$	$\#N/A$
					$\#N/A$	$\#N/A$
						$\#N/A$

Figure 6

True-Objective Probability Measure Option Value Process Evolution

(total probabilities of each outcome are listed under the outcome in light grey.)

0	0.125	0.25	0.375	0.5	0.625	0.75
$q=51.77\% = (\exp((\alpha + r - y) * h) - d) / (u - d)$ $Cu^n d^m = \exp(-ru^n d^{m * h}) * (q * Cu^{n+1} d^m + (1 - q) * Cu^n d^{m+1})$						Cu6= 52.85=Max(Su6-X,0)
				Cu4= 32.28	Cu5= 42.15	1.93%
			Cu3= 23.19	7.19%	Cu4d= 23.48	Cu5d= 32.69=Max(Su5d-X,0)
		Cu2= 15.74	13.88%	Cu3d= 15.00	17.33%	Cu4d2= 15.19Max(Su4d2-X,0)
	Cu= 10.20	26.81%	Cu2d= 3.49	26.77%	Cu3d2= 7.28	25.07%
C0= 6.377	51.77%	Cud= 5.17	38.78%	Cu2d2= 3.49	32.28%	Cu3d3= 0.00Max(Su3d3-X,0)
	Cd= 2.89	49.94%	Cud2= 0.00	37.41%	Cu2d3= 0.00	31.13%
	48.23%	Cd2= 0.80	36.12%	Cud2= 0.00	30.07%	Cu2d4= 0.00Max(Su2d4-X,0)
		23.26%	Cd3= 0.00	23.23%	Cud4= 0.00	21.75%
			11.22%	Cd4= 0.00	14.00%	Cud5= 0.00=Max(Sud5-X,0)
				23.23%	Cd5= 0.00	8.10%
					2.61%	Cd6= 0.00=Max(Sd6-X,0)
						1.26%