

## **Dilution and Multiple-Issue Tranches Inherent in Employee Stock Option Valuation**

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Nearly all publicly traded firms issue employee stock options. This work examines two important characteristics of employee stock options: dilution and the existence of multiple option issues over time. Dilution of share values occurs with employee option exercise (e.g. when an employee actually buys their firm's shares.) Both existing employee option analysis and current reporting rules – FASB 123 – have failed to address employee stock option dilution. Incremental issues occur when firms make additional option grants; and these issues, like dilution, are a pervasive characteristic of employee option valuation.

Since the two mentioned activities affect option values, we analyze their impact in particular cases. Our main task is to discern how the current “Black-Scholes” based standard performs in valuing these options in the face of changing firm ownership structure and value. We evaluate the well-known Galai-Schneller (1978) warrant-based alternative to the Black-Scholes option model. We also develop a new two maturity and strike price warrant pricing model. In comparing the standard model to these two warrant-based alternatives, we find that the Black-Scholes model does quite well. Among the standard option pricing factors, dividend yield is the only factor that differentiates the Black-Scholes approach from the dilution-based method. Furthermore, only dilution changes generate marginally significant value differences between the Black-Scholes and warrant-based models.

Different option issuance tranches can, however, result in significant aggregate employee option values effects. Firms that have significantly different exercise proceeds across option tranches do not accurately reflect the employee option's total equity claim by reporting only aggregate numbers.

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Nearly all publicly traded firms issue employee stock options. This work examines two important characteristics of employee stock options: dilution and the existence of multiple option issues over time. Dilution of share values occurs with employee option exercise (e.g. when an employee actually buys their firm's shares.) Both existing employee option analysis and current reporting rules – FASB 123 – have failed to address employee stock option dilution. Incremental issues occur when firms make additional option grants; and these issues, like dilution, are a pervasive characteristic of employee option valuation.

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### **1. Introduction**

Previous studies (e.g., Marquadt (1999), Core-Guay (2000), Frye (1999), Heath, Hudart-Lang (1999)) have estimated the value of employee stock options. These authors use the standard

Black-Scholes option-pricing model (BS), in which the stock price is the underlying asset.<sup>1</sup> Though this approach is useful and required by the Financial Accounting Standards Board (FASB), the method does not recognize that firms issue employee stock options (unlike exchange-listed options). Consequently, employee option exercise increases both firm value and the number of shares outstanding. Therefore, employee stock options are analogous to warrants.

The option pricing literature (e.g., Galai-Schneller (1978), Lauterbach-Schultz (1990) and Schultz-Trautman (1995) has long recognized that warrants should be valued differently from standard options. With warrants, the underlying asset is not the stock price but the total equity price; that is, the stock price plus the warrant price.

Relative to the standard Black-Scholes option model, warrant valuation requires two simple adjustments to the standard option Model: 1) share dilution and exercise cash receipt on exercise and 2) use of total equity value and its volatility as model inputs. When warrants and/or analogous claims (like employee options) exist, neither the volatility of the traded stock return nor the implied volatility of traded equity options should be used directly as BS option model inputs. We will refer to this standard dilution- and volatility-adjusted method as the warrant-version of BS, or W.

Because employee stock options are not publicly traded, the underlying asset that is required to value these options is not directly observable. Therefore, there are no observable historical returns on the total equity price. Nevertheless, the total equity price and the volatility of its return can be inferred from the employee option values and the relationship between the stock return volatility and the volatility of the total equity price return.

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<sup>1</sup> Appendix A reports the average properties of our sample.

## 2. Data and Model

Our primary data source is the U.S. SEC Edgar 10-K database. In each annual 10-K filing, summary statistics on a firm's employee option issues is provided. We have obtained comprehensive fiscal year-end data on the 1766 firms that entered three S&P major market indices during the 1999 calendar year: 500, Mid-Cap 500 and Small-Cap 500.

This data is the following:

n	outstanding equity shares
S	total traded firm stock value
$\sigma_S$	average volatility of stock value over expected option life
X	total exercise payment made on employee option exercise
m	employee options outstanding
$\bar{t}$	average life – maturity of all employee options
$\underline{t}$	expected life – maturity of newly issued employee options
t	minimum of average and expected lives
y	equity dividend yield to maturity t
r	riskless rate to maturity t

Additionally, we have directly collected information on the breakdown of employee option issues across strike price and maturity classes. For the 1999 sample, we have disaggregated data for 415 firms in the S&P 1500.

Our model development follows the standard option-pricing paradigm:

V	total equity value of the firm, $\frac{dV}{V} = \alpha dt + \sigma dz$ , = S+E
$\sigma_V$	volatility of equity value
E	total employee option value
S	total traded (residual) firm stock value = V-E

Our main modeling approximation is that all employee options mature at the minimum of the average life or the expected life. Given the cumulative nature of reported employee option information, this assumption is, basically, required at some level of aggregation. Our extended sample of 415 firms' information on employee option maturity and strike price classes will permit us

to examine this assumption. Unfortunately, there seems to be little structure in the pattern of reporting across firms. Generally and independent of the extent of employee option issue, three to five exercise price and average maturity classes are reported.

We believe that our setting employee option maturity to the minimum of average life or expected life of the current year issues will permit a first-order approximation to employee option values in the face of two severe complications: the behavioral aspects of employee option exercise decisions and the equity ownership dilution inherent in the existence and exercise of employee options.

The executive option work of Carpenter (1998), Detemple-Sundaresan (1999) and Carr-Linetsky (1999) has, effectively, analyzed the expected maturity approximation of executive option exercise behavior. Using the Black-Scholes model as an alternative to their fuller modeling of exercise behavior, they find that the simple expected maturity approximation is surprisingly effective in replicating option valuation that incorporates broader evaluation of early exercise experience. Our hope is that this observation will carry over to employee options with their potential for large dilution effects.<sup>2</sup>

While these authors' no dilution approximation is clearly reasonable, in most cases, for executive options, the same cannot be said in the employee option case. For fiscal-year-end 1999, outstanding employee options are on average 8.0% of outstanding common equity shares. The first

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<sup>2</sup> Bettis, Bizjak and Lemmon (2002) examine a broad sample of executive option experience, and find that a Carpenter's (1998) simple exercise hazard specification does not perform well relative to her fuller specification of optimal option exercise. They identify the high dividend yield attribute of Carpenter's sample as the main reason for their different finding. With regard to error in the Black-Scholes-expected maturity approximation, they find that it is somewhat greater than documented by Carpenter (1998), approximately 10% on average. Unlike the executive option history, observations of longitudinal exercise experience that are needed to implement the optimal exercise model are not available for the broad class of employee options.

decile runs from a maximum of 738% (Symantec) down to 17.7%.

It is well known (e.g. Galai-Schneller (1978), Emanuel (1983), Cox-Rubinstein (1985), Lauterbach-Schultz (1990), Schultz-Trautman (1994), Crouhy-Galai (1994)) that dilution affects equity warrant values. Except for their vesting and other behavioral characteristics, outstanding employee options also result in dilution of the common equity ownership share. Our work follows the warrant dilution track of research under the simplest assumptions.

Based both on the limited availability of actual employee option exercise information and due to the great complexities of modeling the optimal exercise decisions of employee option holders in the presence of dilution, we use the minimum of expected life and average maturity assumption of the executive option literature to vastly simplify optimal exercise analysis and, effectively, focus on warrant-like dilution effects of employee option issuance. Clearly, and as suggested in Carpenter's conclusion, a full analysis of the actuarial bases of employee exercise behavior coupled with modeling of both dilution and other capital structure issues in the context of corporate claimants' optimal decisions will be a worthwhile (though heroic) effort. Our examination of the 415 firms with partially disaggregated exercise and maturity class data is a start in this direction.

Following Cox-Rubinstein (page 394, Table 7-13), employee option values are a dilution-weighted percentage of standard calls:

$$W = \frac{m}{m+n} C(V, X, t)$$

The Black-Scholes option model with the Merton dividend yield adjustment is the following:

$$C(V, X, t) = Ve^{-yt} N(d_1) - Xe^{-rt} N(d_1 - \sigma\sqrt{t}), \quad d_1 = \frac{[\ln(V/X) + (r - y + \sigma^2/2)t]}{\sigma\sqrt{t}}$$

The value of the firm's equity is the sum of traded equity and the employee stock options,

$V = S + W$ . Hence,  $S = V - W$ . Therefore, the derivative of traded equity is the following:

$$\frac{\partial S}{\partial V} = 1 - \frac{\partial W}{\partial V} = 1 - \left( \frac{m}{n+m} \right) e^{-yt} N(d_1)$$

Among firm equity value,  $V$ ; traded equity,  $S$ ; and employee option values,  $W$ , only the volatility of traded equity is observable. However, the necessary volatility input of employee option valuation is firm total equity value.

Fortuitously, we can infer total equity volatility from traded equity volatility by the following relationship (Cox-Rubinstein, page 187):

$$\sigma_s = \sigma \Omega_{sv} = \sigma \left[ \frac{\partial S/S}{\partial V/V} \right] = \sigma \frac{V}{S} \left[ 1 - \left( \frac{m}{n+m} \right) e^{-yt} N(d_1) \right]^3$$

To estimate the missing firm equity value,  $V$ , and its associated volatility,  $\sigma$ , we use the reported FAS-123 reported estimates for 1999,  $\sigma_s$ , and fiscal year-end traded firm equity value,  $S$ . We utilize the Mathematica FindRoot function to calculate  $\hat{V}$  and  $\hat{\sigma}$  in the following two equation system.

$$S = \hat{V} - \hat{W} = \hat{V} - \frac{m}{m+n} C(\hat{V}, X, t, \hat{\sigma}, \bullet),$$

$$\sigma_s = \hat{\sigma} \frac{\hat{V}}{S} \left[ 1 - \left( \frac{m}{n+m} \right) e^{-yt} N(d_1) \right], \quad d_1 = \frac{\left[ \ln(\hat{V}/X) + (r-y+\hat{\sigma}^2/2)t \right]}{\hat{\sigma}\sqrt{t}}$$

Among 1766 firms that entered the S&P 1500 during 1999, 52 have no employee options outstanding. 21 firms have no reported option maturity, 22 firms have no volatility information, and 14 firms have no reported strike price. Despite the nonlinear aspect of this optimization exercise,

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3. Galai-Schneller, Lauterbach-Schultz and Schultz-Trautman derive warrant pricing relations on a per share basis. From Cox-Rubinstein, the total value specification follows directly with lower case letters representing the per share values of their capital letter analogues:

$$\left( 1 + \frac{m}{n} \right) w = c, \quad w = \left( \frac{n}{n+m} \right) c, \quad w = \left( \frac{1}{n+m} \right) C, \quad \text{and} \quad W = m w = \left( \frac{m}{n+m} \right) C.$$

we had very good convergence results. Only six firms failed to converge in estimation of firm value and its standard deviation.

### **3. Comparison of the Black-Scholes (BS) and Warrant-Based (W) Employee Option Models**

Our first result indicates that the FAS-123 simplifying assumption is not a bad one. Specifically and in terms of the aggregated information at the firm level, the differences between the warrant-adjusted (W) and Black-Scholes (BS) model are relatively small. Figure 1 presents these observations.

In the percentage terms graph, one firm has zero option values and is omitted from the graph. Among absolute value errors, there are 15 errors greater than \$0.50.

Over three quarters of the sample have percentage value differences of less than one percent. Only three percentage value differences exceed 10 percent, and all but 16 are below five percent. The maximum value difference is 17 percent.

The average warrant-based option value is roughly \$20. In over two-thirds of the sample, the price differences are less than ten cents. All but 15 of the price differences are less than \$0.50, five prices differences are greater than \$1.00, and the maximum price difference is \$3.36.

To further emphasize the limited difference between the two models, Figure 2 depicts the average employee option values for both BS and W, and the average differences between the values. Analysis of employee option parameter changes follows Black-Scholes comparative statics analysis. Increases in factors that raise firm value or the value of the employees' share of the firm increase employee option value:  $V$  increase,  $X$  decrease,  $m$  increase. Factors that increase option values also increase employee option values:  $\sigma_V$  increase,  $r$  increase,  $y$  decrease and  $t$  increase ( $r$  is

generally less than  $y$ ).

Except for the yield parameter, the differences between Black-Scholes (BS) and warrant-based (W) employee option values have the same sign as the employee option valuation effects. These smaller (second-order) effects result from the option on an option characteristic of the employee stock options. Employee option holders must share some of their gains with common shareholders. Hence changes in the employee option value functions are more damped than in the Black-Scholes case. Yield increase-induced differences in BS and W values are positive, but humped. The more complicated yield phenomenon results from the joint impact of yield on the employee options value and the firm variance through delta.

Though all parameters affect delta and, hence, impact firm value variance, firm value and yield have multiple impacts. For yield, the negative impact of discounting the normal distribution component of the variance equation dominates its delta effect. Therefore, increased yield lowers both BS and W values directly, and the additional negative variance impact of yield changes makes warrant-based values more sensitive to yield than Black-Scholes values.

Table 1 reports regression analysis of cross-sectional percentage valuation differences between the Black-Scholes and warrant models. Among the standard option model inputs, all factors other than volatility manifest the same impact as exhibited in our comparative statics analysis.

Particularly, the positive firm value and negative strike price impacts on Black-Scholes overvaluation relative to the warrant-based model are offsetting in the cross-section. While volatility increases raise the difference between Black-Scholes and warrant values in comparative statics analysis, the cross-section regression indicates that higher volatility firm employee options are less overvalued by Black-Scholes relative to the warrant-based model (all else equal.)

Firms with longer maturity options face larger Black-Scholes model biases. The most significant value differences arise for dividend yield. Higher dividend yield firms have significantly over-valued options using Black-Scholes relative to the warrant-based alternative. Finally, only a very insignificant interest rate effect is estimated.

#### **4. Quality of Aggregate Employee Option Value Information**

Given the relatively small average model value differences, the general features of the sample may be represented by the Black-Scholes (BS) Model as proscribed by FAS-123. In addition, the Black-Scholes Model has another useful feature. Since firms issue employee options in different exercise and maturity tranches, separate valuation of these tranches is straight-forward with the Black-Scholes model. In this manner, we may see whether aggregation of employee option data into firm-wide totals and averages results in a biased picture of employee's option claim on firm value.

As noted in our data discussion, we have compiled disaggregated employee option information for 415 of the 1651 firms in our estimation sample. This information is taken from the annual SEC Edgar 10-K filings that are in standard html format, and is for firms with differences between pro-forma income and reported income.<sup>4</sup>

Figure 3 breaks down the firms by the number of reported option tranches. The mean and median number of tranches is five, and the maximum is 50. Interestingly, the firm reporting 50 tranches is a small firm, Digital Lightwave; and, apparently, this firm reports all issues separately. Among the firms with the most valuable employee option issues, only a limited number of tranches

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<sup>4</sup>Under FAS 123, firms must report earnings per share adjusted for outstanding options that are in-the-money as pro-forma earnings per share. Firms with moderately significant employee option values will

are reported. Among our 415 firm sample, Microsoft has the largest aggregate employee option values, and reports only six tranches.

The usual method of reporting employee option values and implications for changes in their terms, regulation and reporting has been to analyze the average characteristics of the option issues across all issue tranches. We have followed this precedence in our analysis.

To proceed in this evaluation, we simply aggregate the firm data into two tranches based on maturity. Across option tranches, it is maturity differences that manifest the most interesting and complicated valuation issues. Exercise price and grant amount differences are also important. Nevertheless, the correlation between the maturity and strike price ordering is over 84%. This ordering results predominantly from the 1990's bull market and the associated growth of employee option issue over this decade.

To construct two option grant tranches for each sample firm, reported option grants are ranked by maturity, and then aggregated by outstanding amounts into two equal-sized groups. The option terms for each separate tranche are weighted by relative outstanding amount: i.e. exercise price and maturity. We identify the tranches as either short-term (s/t) or long-term (l/t). In order to provide initial results on relative employee option values, we also aggregate across all firms to create a representative firm with two employee option issue tranches.

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manifest a measured difference between earnings per share and pro-forma earnings per share.  
(Compustat source)

Across the 415 firms with disaggregated data, the average firm has the following characteristics:<sup>5</sup>

# shares	178,518,223
Share price	\$37.60
Total share value	\$9,231,852,920
# options	16,347,513
# tranches	5.02
Discount rate	5.71%
Volatility	45.04%
Yield	0.75%
# s/t options	6,074,648
s/t option maturity	5.46
s/t option exercise price	15.09
Value s/t options	\$361,525,019
# l/t options	6,480,373
l/t option maturity	8.07
l/t option exercise price	27.90
Value l/t options	\$253,654,862
Value s/t and l/t options	\$615,179,880
ave. option maturity	6.83
ave. option exercise price	21.31
Value ave. option tranche	\$602,749,285

The option values are calculated with the Black-Scholes model using share price inputs and do not incorporate dilution. In general, these average characteristics are indicative of the sample. However, three notable exceptions exist: Intel, Comerica and Pfizer.

Intel presents the largest value difference between the aggregate method relative to the two tranche method: \$610,395,970. Nevertheless, this difference is only 3.15% of approximately \$19.5 billion in outstanding Intel employee options.

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<sup>5</sup> We follow the general valuation practice and assume that all warrant holders in a class act in unison and that the firm uses any warrant exercise proceeds so that the current share price is unchanged. For our sample, the two maturity tranches are treated separately because the longer-maturity option holders are better off waiting to contract maturity rather than exercising with the shorter-maturity option holders. Specifically, exercising the long-term options early (with the first tranche options) results in a value of only \$132,628,168; relative to the full term value of \$144,444,048. Full evaluation of multiple classes of warrants is a very complicated problem. For discussion of single warrant issue valuation, see Cox-Rubinstein (1985, pp. 397-399), Emanuel (1982), Constantinedes-Rosenthal (1984), and Spatt-Sterbenz (1988). Kapadia-Willette (2001) treats multiple classes of warrants in a general setting.

Comerica has the largest percentage valuation difference of 25.2%. Their short-term tranche maturity is 4.48 years and the associated strike price is \$20.25. The long-term tranche maturity is 8.29 years and strike price is \$60.62. The short-term and long-term tranche Black-Scholes values are \$96,119,851 and \$47,418,045, respectively; and \$143,537,897 in total. Aggregated over these two tranches by number of options outstanding, maturity is 6.73 years, exercise price is \$44.12, and option value is only \$114,643,530, 25.2% less than in the disaggregated case.

Pfizer has the second largest value difference between the two methods, \$531,201,395. The percentage difference is also relatively large, 12.33%. Their short-term tranche maturity is 5.15 years and the associated strike price is \$9.5. The long-term tranche maturity is 8.82 years and strike price is \$36.4. The short-term and long-term tranche Black-Scholes values are \$3,285,852,840 and \$1,554,818,516, respectively, and \$4,840,671,357 in total. Aggregated over these two tranches by number of options outstanding, maturity is 6.94 years, exercise price is \$22.63, and option value is only \$4,309,469,961, 12.33% less than in the disaggregated case.

We will now investigate if treating dilution in the case of two option tranches results in significantly different values than these Black-Scholes benchmarks. The model is a recursive dynamic programming application with integer variables. We will find that like the aggregate case, the value difference is not large.

## **5. Dilution of Equity Value with Two Distinct Employee Option Issue Tranches**

In this section, we extend the standard binomial option pricing methodology to treat both dilution and conditional European option exercise over two dates and two sets of options. To our

knowledge this valuation approach is new. Furthermore, the work provides only a first step in valuing multiple tranches of employee options. Complexity in this analysis results because each set of outstanding option holders must react to three contingency sets: potential actions of all holders, the firm's use of exercise proceeds, and changes in their own idiosyncratic circumstances.

These situations are well known as cases that manifest strategic warrant exercise, exercise proceeds use questions, and hazard attributes, respectively. However, we abstract from gaming complexities and follow the suggestion of that literature: we treat all members of a warrant class in a common manner. Regarding use of option exercise proceeds, we assume that any exercise proceeds are distributed to shareholders or invested in the firm so as to leave the firm share price unchanged. With regard to hazard-related conditions such as vesting, premature exercise and termination, we conjecture that the relative nature of our tests that compare Black-Scholes and the directly-related dilution alternative will be indicative of the scale of any differences for broader specifications.

In our model, we have two outstanding employee option issues or tranches. The tranches are differentiated from the average maturity tranche case as follows.

- $t_k$  average life – maturity of all employee options in tranche  $k$ ,  $k=1,2$
- ${}_1t_2$  average life – maturity between tranche maturities,  ${}_1t_2 = t_2 - t_1$
- $X_k$  total exercise payment made on tranche  $k$  employee option exercise,  $k=1,2$
- $m_k$  employee options outstanding in tranche  $k$ ,  $k=1,2$
- $n_k$  number of discrete time steps in tranche maturity  $k$ ,  $k=1,2$
- $h_k$  time step in tranche maturity  $k$ ,  $k=1,2$ ,  $h_k = t_k / n_k$
- ${}_1n_2$  number of discrete time steps in between tranche maturities,  ${}_1n_2 = n_2 - n_1$
- ${}_1h_2$  time step in tranche maturity  ${}_1t_2$ ,  ${}_1h_2 = {}_1t_2 / {}_1n_2$

Our model development follows the binomial method of Amin (1991), as extended in Amin-Bodurtha (1995). We define the up and down outcomes as functions of the risk – neutral

drift, volatility and time step length. Also, we choose the number of time steps in each tranche so that the time step lengths are equal in each maturity tranche ( $h = h_1 = h_2$ ).

$$\alpha = -\text{Ln} \left( \frac{e^{s\sqrt{h}} + e^{-s\sqrt{h}}}{2} \right) \text{ the risk-neutral drift.}$$

$$u = e^{\alpha + (r-y)h + s\sqrt{h}} \text{ the proportional binomial up jump}$$

$$d = e^{\alpha + (r-y)h - s\sqrt{h}} \text{ the proportional binomial down jump}$$

$$V_1(i) = V_0 n u^{n_1-i} d^i, i = 0, n_1 \text{ firm value outcomes at maturity } t_1$$

$$V_2(i) = V_0 n u^{n_2-i} d^i, i = 0, n_2 \text{ firm value outcomes at maturity } t_2$$

$$p_k(j) \text{ binomial probability of } j \text{ successes in } n_k \text{ equal chance trials}$$

$${}_1p_2(j) \text{ binomial probability of } j \text{ successes in } {}_1n_2 \text{ equal chance trials}$$

As noted above, the two tranche employee valuation problem is tractable because the optimal exercise decision by one tranche permits conditioning on this decision by option holders in the second tranche. For the shorter maturity tranche, the boundary condition that identifies the second tranche holders' exercise conditional on the first tranche holders' exercise is

$$z_2(i) = 1 \mid V_2(i) / X_2 > n + m_1, \text{ and zero otherwise}$$

Should the second tranche of employee option holders exercise, then the first tranche of holders will receive a relatively smaller fraction of firm value, but will also receive a share of the second tranche exercise proceeds. The cashflow accruing to the first tranche of employee option holders at maturity of the second tranche is conditional on the second tranche's exercise behavior:

$${}_1w_2(i) = (V_2(i) + z_2(i)m_2X_2) / (n + m_1 + z_2(i)m_2)$$

Contingent on firm value outcomes at time  $t_1$ , the values and associated boundary conditions for the first tranche of employee options are defined:

$$w_1(i) = \text{Max} \left[ \sum_{j=0}^{n_1} p_1(j) w_2(j) e^{-r_1 t_1} + \frac{m_1 X_1 + V_1(i)(1 - e^{-y_1 t_1})}{n + m_1} - X_1, 0 \right] \geq 0, i = 0, \dots, n_1$$

This type of boundary condition greatly simplifies analysis and computation in the two option issue tranche case. Given our assumptions of a fixed firm value process and all options in a class being exercised simultaneously, the first tranche option holders are able to evaluate both their own optimal policy and that of the second tranche option holders conditional on their own exercise decision.

The current value of this employee option is the following:

$$w_1 = \sum_{j=0}^{n_1} p_1(j) w_1(j) e^{-r_1 t_1}$$

The second option tranche value and exercise decision is conditional on the first tranche holders' optimal exercise at maturity:  $z_1(i) = 1 | C_1(i) > 0$ , and zero otherwise. Conditional on this exercise indicator, the second tranche of employee option holders' firm value share may be calculated at maturity:  ${}_1V_2(i) = V_1(i)/(n + z_1(i)m_1)$ ,  $i = 0, \dots, n_1$ . The end of period  $t_1$  option value is  $w_2(i) = C({}_1V_2(i), X_{2,1}t_2) \geq 0$ ,  $i = 0, \dots, n_1$ .

Finally, the value of the second maturity tranche employee option tranche is

$$w_2 = \sum_{j=0}^{n_1} p_1(j) w_c(j) e^{-r_1 t_1}$$

As in our previous analysis, we must infer the value of these options based on observed equity value and volatility. For average equity value over our average firm observations, we will infer firm value and volatility from the following relations,

$$S = \hat{V} - \hat{W}_1 - \hat{W}_2 = \hat{V} - m_1 \hat{w}_1(\hat{V}, X_1, t_1, X_2, t_2, m_1, m_2, \hat{\sigma}, \bullet) - m_2 \hat{w}_2(\hat{V}, X_1, t_1, X_2, t_2, m_1, m_2, \hat{\sigma}, \bullet)$$

$$\sigma_s = \hat{\sigma} \frac{\hat{V}}{S} [1 - \hat{\Delta}_1 - \hat{\Delta}_2], \quad \hat{\Delta}_k = \frac{w_k^u - w_k^d}{V_0^u - V_0^d}$$

Using the binomial approximation, the option deltas are calculated with one-step ahead option and firm values:

$$w_k^u = \sum_{j=1}^{n_1} p_1^u(j) w_k(j) e^{-r(t_1-h)}, \quad w_k^d = \sum_{j=0}^{n_1-1} p_1^d(j) w_k(j) e^{-r(t_1-h)}$$

$$V_0^u = u V_0, \quad V_0^d = d V_0$$

$p_1^u(j)$  is the binomial probability of  $j$  successes in  $n_1-1$  trials with equal chance of success or failure.

As done previously for the case of one aggregate option class, we analyze the sensitivity of the Black-Scholes model and our two tranche option model to parameter changes. Unlike the previous analysis in which each individual issue was perturbed by the parameter change and then aggregated, we examine the impact of parameter changes on the average firm only.<sup>6</sup>

Figure 4 presents the results. The most striking feature of these results is, again, the small relative value differences between approaches. For most option pricing factors, the value differences between the simple Black-Scholes approach and our conditional two tranche option approach are less than three percent. Except for the relative size of the two option tranches, the Black-Scholes approach over values the shorter maturity options and under values the longer

maturity options. For the relative tranche size case, a relatively small initial maturity tranche results in the standard pattern of Black-Scholes over valuation of the first tranche and under valuation of the second tranche. Moreover as the size of the first tranche grows, the pricing bias is reversed for the shorter term options and the value difference approaches zero for the longer term options.

The response of the two tranches to increases in the option factors also differ. The following table compares the results in the disaggregated case to the Section 3. and Figure 2. aggregated results. For dilution, exercise price and time to maturity factors, the relative size of the first maturity tranche factor is changed:

Impact on Difference Between Black-Scholes and Warrant-Based Model Values

Increase in factor	Aggregated Tranches	First Tranche	Second Tranche
Total equity value ( $v$ )	Increasing	Mildly increasing	Decreasing
Volatility ( $\sigma_v$ )	Increasing	Mildly increasing	Decreasing
Exercise price ( $x, x_1, x_2$ )	Decreasing	Mildly increasing	Increasing/decreasing
Dilution ( $m, m_1, m_2$ )	Decreasing	Increasing/decreasing	Increasing
Time to maturity ( $t, t_1, t_2$ )	Increasing	Mildly increasing	Mildly decreasing
Yield ( $y$ )	Increasing/decreasing	Increasing/decreasing	Increasing
Rate ( $R$ )	Increasing	Constant	Decreasing

For the average/representative firm case that we have analyzed, the valuation effects for the shorter maturity (and lower exercise price) tranche are more similar to the single tranche and standard warrant based model biases than the longer-term options.

## 6. Disaggregated Employee Option Grant Information

Generally, our results indicate that the Black-Scholes modeling approach gives a reasonable approximation to employee option values. With this approximation, we evaluate whether aggregate reported employee option data provides sufficient information about the

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<sup>6</sup> Treating the average firm in the single employee option tranche case yielded results similar to those presented. We conjecture that the same situation will prevail in the two employee option tranche case.

value of employee option claims.

Figure 5 presents a histogram of the percentage mis-valuation of one aggregate option issue tranche versus two disaggregated issue tranches. By treating all of a firm's employee options as one aggregate issue, the average under valuation of the equity claim is over five percent. From Figure 5, we also see that significant differences exist for a fair number of firms. Rudimentary regression analysis shows that these differences are most strongly associated with firms that have significant differences in their exercise proceeds.

This regression test is specified linearly:

$$\begin{aligned} \text{Difference in two tranche value and single aggregated value} = \\ a + b_1 \text{ maturity difference} + b_2 \text{ exercise price difference} \\ + b_3 \text{ outstanding option difference} + b_4 \text{ exercise proceeds difference} + \text{error} \end{aligned}$$

Our regression estimates are presented in Table 2. Both the difference in the number of options and the total exercise proceed difference between the two tranches are significantly and positively related to differences between the disaggregated two tranche value sum and the value of one aggregated tranche. Therefore, it is important to have disaggregated employee option information for firms with grants that have been made at very different exercise prices and/or are of very different size.

## **7. Conclusion**

On first thought, it would seem that FAS-123 mandated use of the simple Black-Scholes for valuing employee option would be problematic. Numerous biases exist in such an application, and we have examined the dilution aspect of this problem in detail. Specifically with regard to dilution, we find that the simple Black-Scholes approach is both reasonably accurate and unbiased.

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We hope to provide aggregated sensitivity results at the individual firm level in subsequent work.

Our conclusion follows from benchmarking the Black-Scholes approach against the Galai-Schneller (1978) model that incorporates dilution of a firm equity with only one class of warrants (employee options) outstanding. We evaluate these two models for the 1999 fiscal year-end sample of all S&P 1500 firms.

Additionally, we develop a NEW two employee option issue tranche binomial model extension to the Galai-Schneller model, and evaluate this model against Black-Scholes. We benchmark the two tranche model with a sample of 415 of the S&P 1500 firms that have multiple tranches of employee options outstanding. Across the seven standard inputs to the warrant valuation problem (firm value, firm return volatility, exercise price, dilution, time to maturity, yield and rate), changing relative dilution is the only factor found to generate greater than five percent differences between Black-Scholes and the models that incorporate dilution.

Of course, our work addresses but one of the significant valuation issues embodied in the employee and the related executive option problems. The works of many have highlighted the multi-dimensional aspects of this problem. Rubinstein (1995) has provided a taxonomy of seven particular valuation issues: maturity, delayed vesting, forfeiture, non-transferability, taxes, capital structure effects, operating income effects. Based in his taxonomy, he identifies nine additional inputs to the standard Galai-Schneller (1978) warrant/employee option valuation approach.

Across these factors, Rubinstein shows that valuation errors on the order of 50% may be generated by relatively small joint perturbations of the valuation parameters. Our specific modeling of the two tranche dilution problem is not alone sufficient to generate such biases. We believe that the main reason that we do not find a significant dilution effect is because we must assume that traded equity values are correct. Otherwise, we have insufficient information from which to

estimate firm value and its volatility. Effectively, we solve an inverse problem with limited information. On the contrary, were we privy to internal corporate information about the value process, dividend payout policy and option grant and vesting conditions, then dilution could matter to a great degree.

With regard to the quality of information regarding dilution, our analysis of the disaggregation of our sample into two tranches shows that significant value differences exist when total exercise proceeds differ across employee option tranches. This observation points directly to the need for disclosure of disaggregated employee option grant data. Going even further, we reassert Carpenter's (1998, pp. 152-3) valuation point regarding the need for longitudinal data on the behavioral aspects of option grant programs. Finally, our result is in line with Rubinstein's (1995) emphasis of the need for better information on the structure of employee option claims on equity value to prevent accounting and value manipulation.<sup>7</sup>

Recent work by Hall and Murphy (1999) emphasizes the inapplicability of the Black-Scholes risk-neutral valuation approach to executive and employee stock options, which are inherently risky to the holder. Their broad conclusions suggest that options are most appropriate as add-ons to pay packages, while restricted stock serves better as a substitute for existing compensation. Our results lead us to suggest that their broad findings should not be changed by the existence of multiple option grants over time and across option strike prices.

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<sup>7</sup> The existence of option and restricted stock plans has led to many related corporate and high-net worth individual hedging transactions. Much like the issue of the firm or insider hedging their market risks, firms and individuals actively manage their stock plan risks with put sales, collars, contingent value right purchases, outright forwards and stock loan-related transactions. For discussion of a possibly egregious example of these activities, see Bodurtha (2002).

## **Appendix A: Descriptive Statistics**

For 1999 fiscal year end, we have created a data set with observations on 2987 firms. Our data set includes the following quantities: Total Equity Value, Stock Price (S), Volatility, Period of Exchange Listing, Maturity, Exercise Price (X), Option In-the-Money percentage (S/X), Dilution, and Employee Option Values.

Our sample is in two segments. The first data set includes all 1651 firms that entered the 1999 S&P 1500: S&P 500, Mid-Cap 500 or Small-Cap 500 . The second data set is a sample of remaining firms that report employee option information on S.E.C. Edgar in standard html format, and had some employee options that were in-the-money (as indicated by the difference between undiluted and diluted earnings in Compustat.) Based both on our modeling results and to be comparable to other employee option empirical work, we have calculated employee option values using the FAS-123 mandated Black-Scholes model framework.

Across our two data sets, the S&P 1500 set has more pricing information. Particularly, the data associated with calculation of the expense of annual employee option grants is available. Therefore, this sample has additional information on expected option maturity, volatility, dividend yield and riskless discount rate from the earnings per share section of the annual 10-K footnotes. For valuing the other firms' employee options, we use average maturity, the average of all reported riskless rates across S&P 1500 firms, a year-end 40 day historical volatility estimate and the year end dividend yield as calculated from CRSP. Benchmarking the impact of these data differences for the S&P 1500 firms yields no significant aggregate differences.

Table A-1 presents the average, median and standard deviation for each of the key

variables. Aggregate and disaggregated results are reported.

On average, the total equity value for our sample firms is nearly \$4.7 billion (median = \$480.6 million). On a per share basis, the average stock price is \$26.09 and the median is \$18.30. The average annual stock return volatility is 64.33 percent and the median volatility of 50 percent. The average dilution ratio of employee stock option to total diluted shares outstanding,  $M/(N+M)$  is 10.9 percent, with a median ratio of 9.1 percent. On average, employee options were in-the-money: 1.78 average stock price to exercise price ratio and 1.16. median ratio.

Employee options represent a small, but not insignificant, component of firm equity value. The average employee option values are about \$299 million and the median is just \$27 million. As fractions of total equity value, the quantities are 6.33 and 5.55 percent, respectively.

Across our two samples, the S&P 1500 firms dominate the other sample in terms of value. However, in terms of relative importance, the other non-S&P 1500 firms are more impacted by employee options. Generally, these firms are smaller and riskier. Employee option dilution and option value as a percent of total equity value are also higher for the second set of firms. Furthermore, and remembering that year-end 1999 was the peak of the 1990's bull market, the average degree that options were in-the-money is greater for the non-S&P 1500 firms than the S&P 1500 firms. However, the median employee option in-the-money ratio is roughly the same across segments.

Clearly, our two data sets represent large- and small-firm samples. To focus on the differences between these samples, Table A-2 reports sample characteristics across CRSP value decile groups. Firms in the largest 10 percent of the CRSP NYSE-Amex-Nasdaq universe have, on average, employee stock option values that are 5 percent of the total common stock value

(median = 2.9 percent). At the other extreme, the smallest 10 percent of the CRSP NYSE- Amex- Nasdaq universe have, on average, employee stock option values that are 21.2% percent of the total common stock value (median = 12.1 percent). Generally, smaller firms have more of their equity value concentrated in employee options.

We can better understand the source of size-related aspects of relative employee option value by reviewing the volatility, maturity, in-the-moneyness and dilution across our two sample segments. The second panel of Table A-2 highlights the significant differences between the S&P 1500 firms and the other firm set. On average, S&P 1500 firms have lower volatility, shorter maturity, are less in-the-money and have less dilutive employee option plans. All of these factors result in the lower relative valuation of S&P 1500 firm options. Finally, the S&P 1500 firms are significantly “older” than the second firm segment of our data set. Overall the lower relative value of the S&P 1500 firm employee options is related to these firm’s seniority and lower risk.

The dilution and volatility features of our sample are presented across industries in in Tables A-3-a and A-3-b. Somewhat surprisingly, Table A-3-a shows that in terms of industry averages, employee option grants do not differ greatly. For S&P 1500 firms, dilution is approximately 5-10%. For the non-S&P 1500 firms, this range is roughly 6-15%. Over a large number of industries, the S&P 1500 dilution levels are significantly less than the other firm sample.

Table A-3-b reports the volatility estimate breakdown across industries. Again, we see that the S&P 1500 firms are less risky than the other firms. The only exception is the building materials, hardware, garden supply industry (#52), which has only six firms.

The correlation coefficients for the primary inputs into the pricing model and some other variables are shown in Table A-4. Firm value ( $V*N$ ) is marginally negatively correlated with

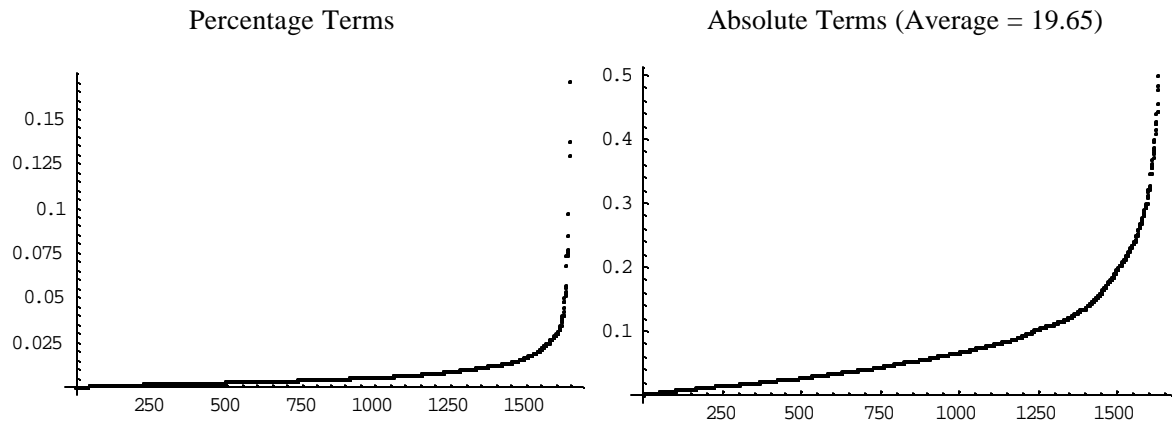
volatility, the degree that options are in the money, and employee options outstanding. Higher volatility is associated with more dilution, shorter-lived equity and lower dividend yield. Dilution is negatively related to period of listing and yield. As noted already, more valuable firms tend to have fewer and less valuable employee option claims on firm equity value.

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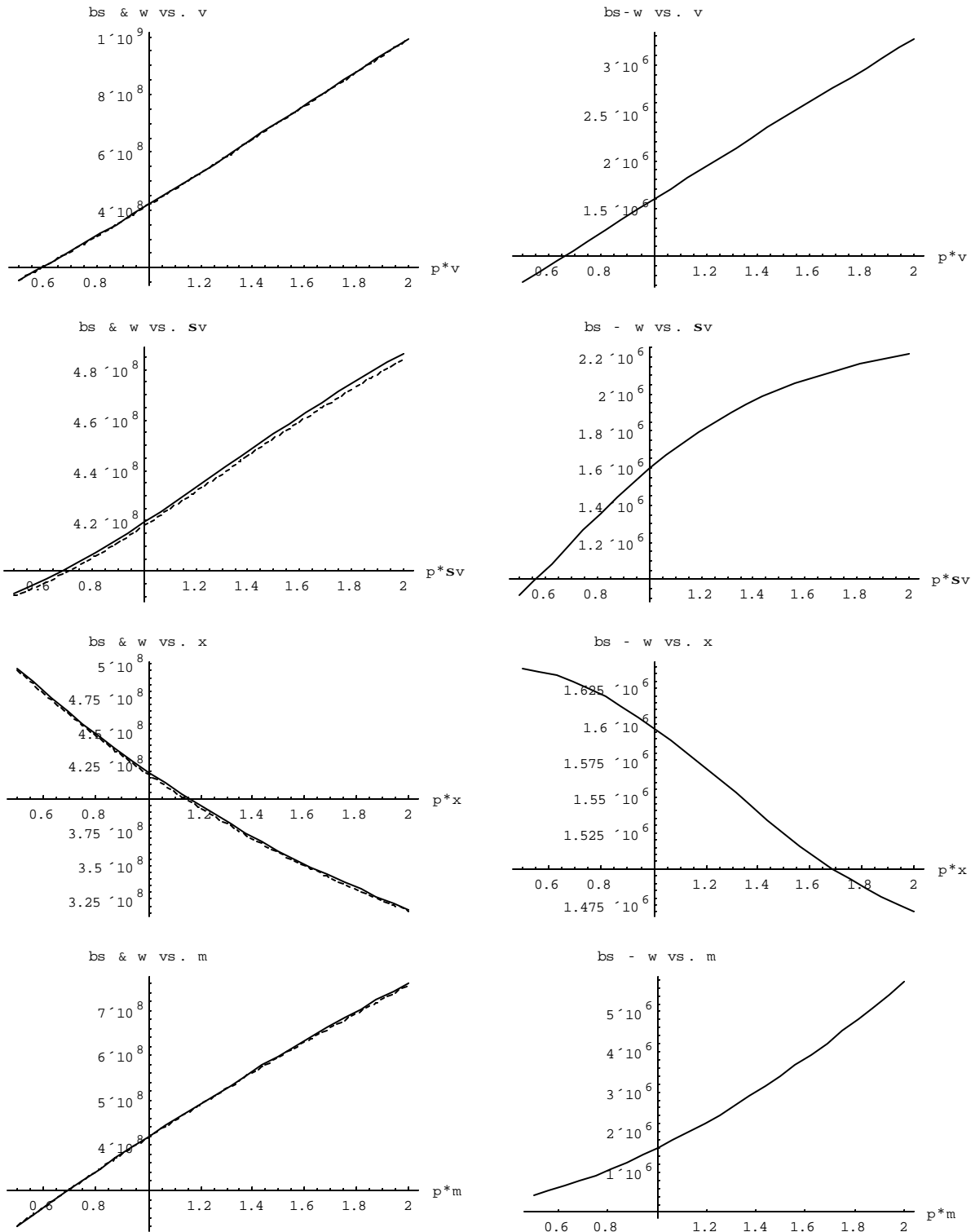
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**Figure 1: Black-Scholes Model Overvaluation relative to the Warrant-Based Model**

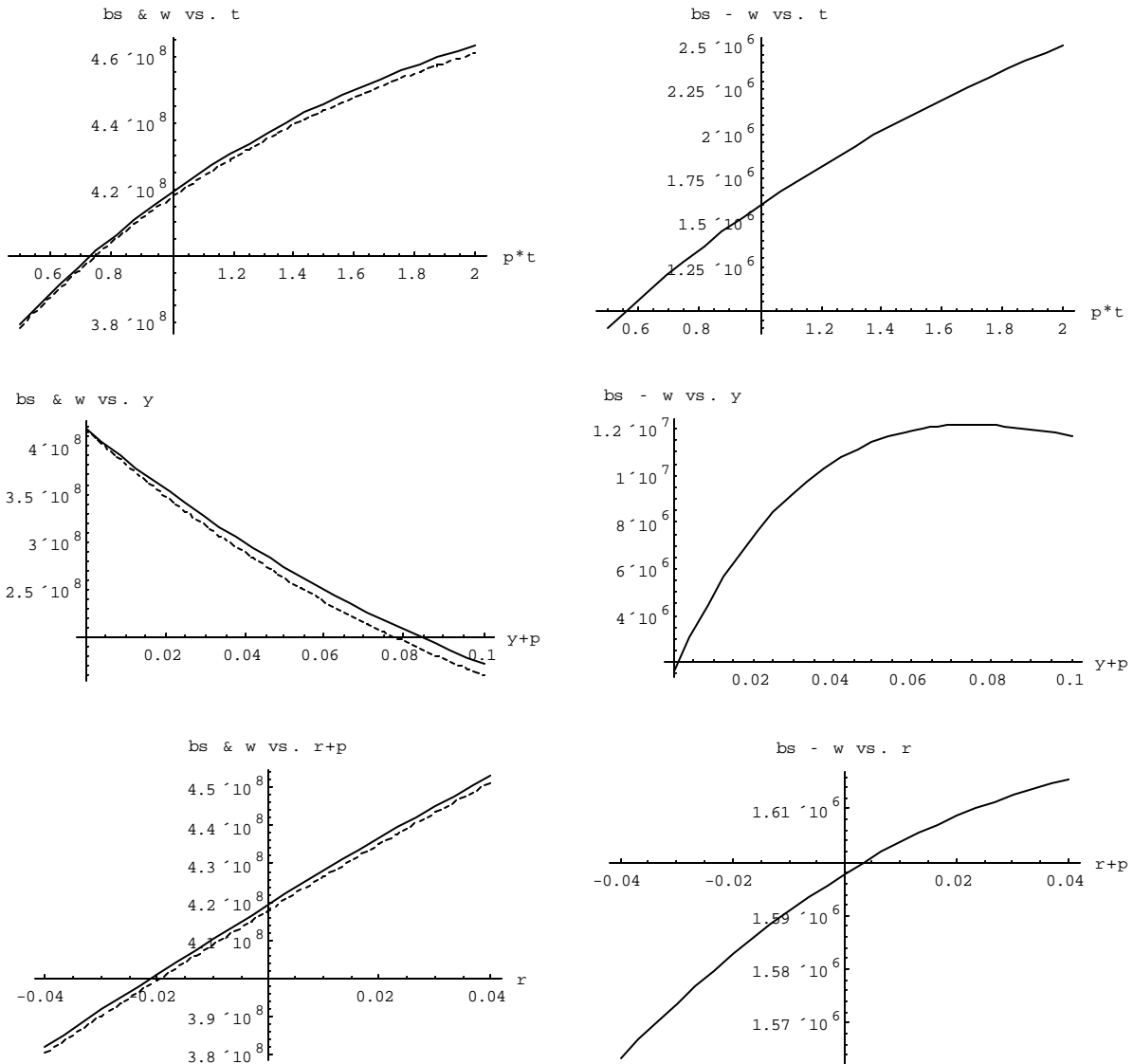


**Figure 2-a: Black-Scholes (BS) and Warrant-based (W) Employee Option Value: Averages and Average Differences with respect to Total Equity Value (v), Total Value Volatility ( $\sigma v$ ), Exercise Price (x), and Dilution (m)**



- As indicated in the axes labels, the difference between warrant-based and Black-Scholes employee option values are plotted with the vertical axis. Importantly, equity value (s) volatility ( $\sigma$ ) are permitted to vary endogenously with the other parameter values. Our sample is the 1999 S&P 1500 firm fiscal year employee option grants.

**Figure 2-b: Black-Scholes (bs) and Warrant-based (eo) Employee Option Value: Averages and Average Differences with respect to Time to Maturity (t), Yield (y), and Rate (r)**



- As indicated in the axes labels, the difference between warrant-based and Black-Scholes employee option values are plotted with the vertical axis. Importantly, equity value (s) volatility ( $\sigma$ ) are permitted to vary endogenously with the other parameter values. Our sample is the 1999 S&P 1500 firm fiscal year employee option grants.

**Figure 3: Separate X/T Employee Option Tranches**

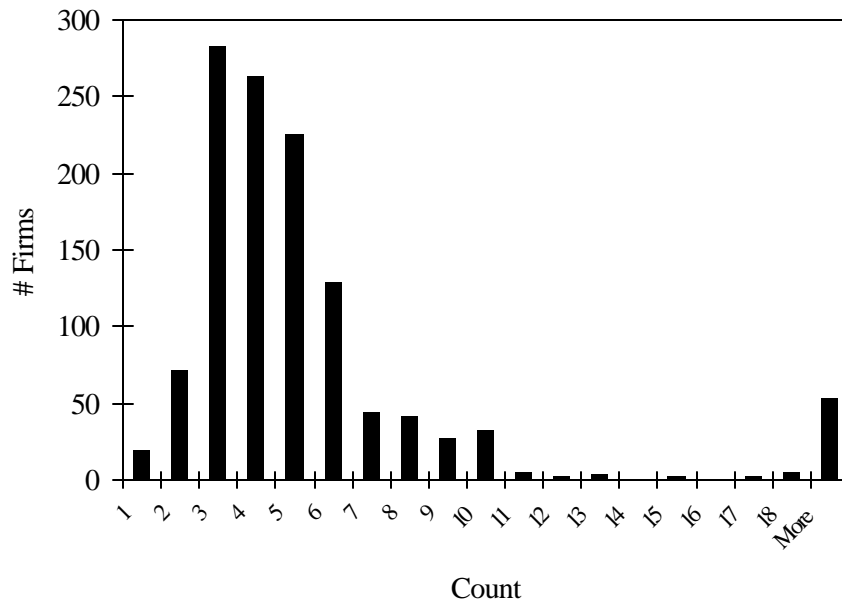
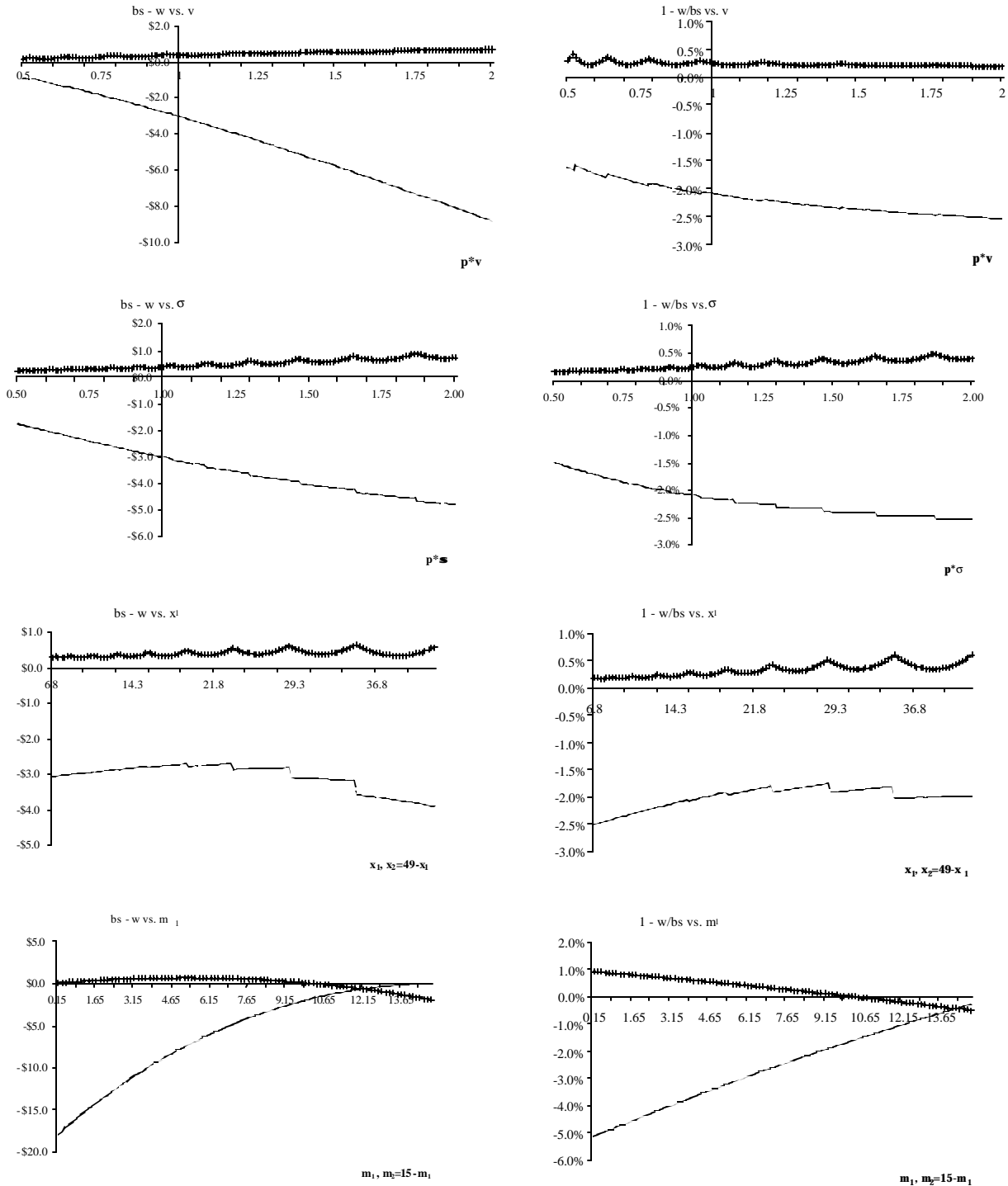
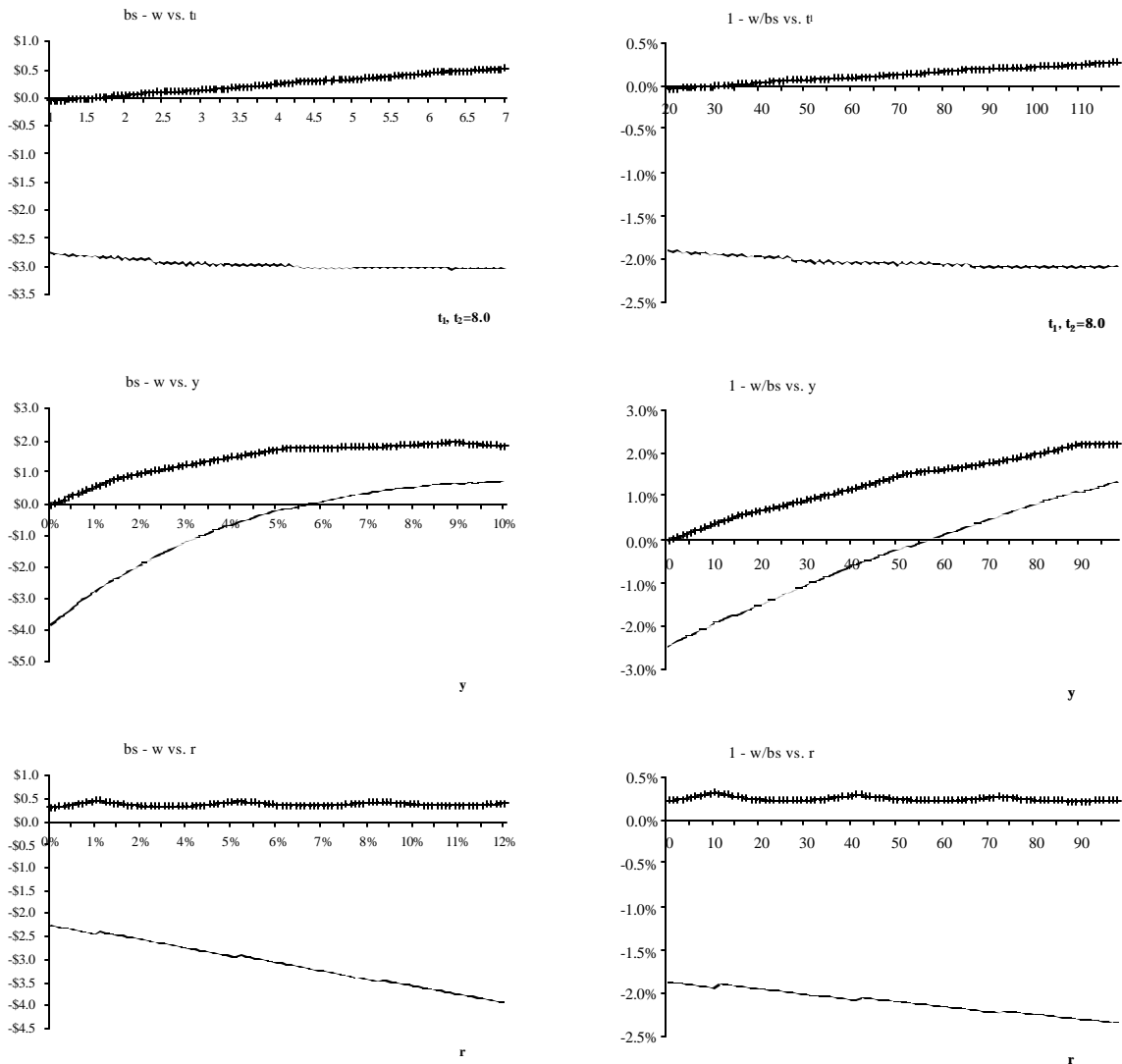


Figure 4-a: Two Tranche Black-Scholes (BS) and Warrant-based (W) Employee Option Values: Per share Average Differences and Percentage Differences with respect to Total Equity Value ( $v$ ), Total Value Volatility ( $\sigma_v$ ), Relative Exercise Prices ( $x_2=x-x_1$ ), and Relative Dilution ( $m_2=m-m_1$ )



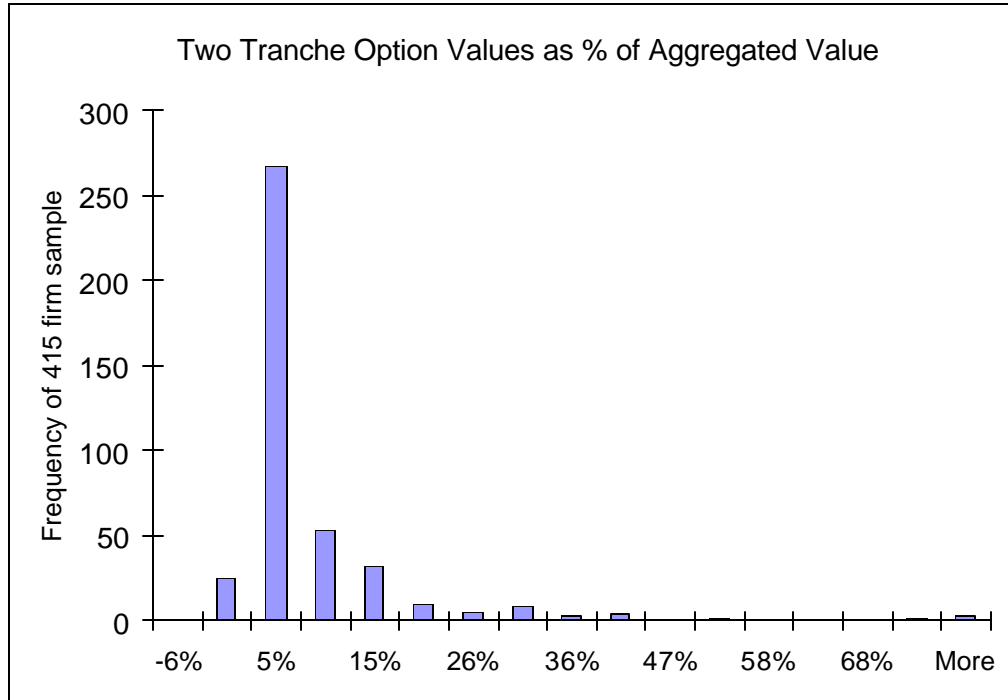
- As indicated in the axes labels, the warrant-based and Black-Scholes differences are depicted for the shorter-maturity tranche 1 (+++) and the longer maturity tranche 2 (---). Per share employee option value differences and percentage differences are plotted with the vertical axis. The benchmark case warrant values are \$27.17 for 6,008,216 first tranche warrants,  $w_1$ , and \$23.54 for 6,258,177 second tranche warrants,  $w_2$ . Importantly, equity value ( $s$ ) volatility ( $\sigma$ ) are permitted to vary endogenously with the other parameter values. Our sample is the 1999 S&P 1500 firm fiscal year employee option grants.

Figure 4-b: Two Tranche Black-Scholes (BS) and Warrant-based (W) Employee Option Values: Per share Average Differences and Percentage Differences with respect to Time to Maturity (t), Yield (y), and Rate (r)



- As indicated in the axes labels, the warrant-based and Black-Scholes differences are depicted for the shorter-maturity tranche 1 (+++) and the longer maturity tranche 2 (-). Per share employee option value differences and percentage differences are plotted with the vertical axis. The benchmark case warrant values are \$27.17 for 6,008,216 first tranche warrants,  $w_1$ , and \$23.54 for 6,258,177 second tranche warrants,  $w_2$ . Importantly, equity value ( $s$ ) volatility ( $\sigma$ ) are permitted to vary endogenously with the other parameter values. Our sample is the 1999 S&P 1500 firm fiscal year employee option grants.

**Figure 5**



**Table 1: Regression Analysis of Percentage Black-Scholes Model Overvaluation Relative to the Warrant-Based Model**

Explanatory Variable	Constant	Coefficient	Adjusted R <sup>2</sup>
Firm Value/Exercise Price	0.0025 (1.13)	0.000 (-1.37)	0.0005
Maturity	0.0025 (1.13)	0.0079 (11.6)	0.0031
Stock Price Volatility	0.0118 (8.25)	-0.0095 (-3.26)	0.0058
Options Outstanding/ Shares Outstanding	0.0067 (9.11)	0.0091 (2.71)	0.0038
Dividend Yield	0.0005 (1.03)	0.5770 (43.1)	0.5310
Interest Rate	-0.0025 (-0.37)	0.1772 (1.51)	0.0008

Estimated for 1655 firms that entered the S&P 1500 during the 1999 calendar year. Cross-section is from 1999 fiscal year 10-K filings.

**Table 2: Regression of Difference between One Tranche and Two Tranche Black-Scholes Employee Option Values on Maturity, Strike Price, Outstanding and Exercise Proceed Differences across Tranches**

Variables	Coefficients	t Stat	Adjusted R Square
Intercept	-470908	-0.216	0.442
maturity difference	-383866	-0.539	
exercise price difference	110387	1.371	
outstanding option difference	0.1793	2.640	
exercise proceeds difference	0.0304	16.976	

Estimated for a 415 firm sample drawn from the 1655 firms that entered the S&P 1500 during the 1999 calendar year. Cross-section is from 1999 fiscal year 10-K filings. Reported option grants are ranked by maturity, and then aggregated by outstanding amounts into two equal sized groups. The option terms for each separate tranche are weighted by relative outstanding amount.

**Table A-1: Sample Averages, Medians and Standard Deviations**

Sample	Statistics	Total Equity Value	Stock Price (S)	Volatility	Period of Listing	Maturity	Exercise Price (X)	S/X	Dilution	Employee Option Values	Employee Options/Equity
2987 firms combined	Average	4,725,840,000	26.09	64.3%	13.47	6.86	17.47	1.78	10.91%	299,026,873	6.33%
	Median	480,601,725	18.30	50.0%	8.40	7.06	14.44	1.16	9.07%	26,695,003	5.55%
	Stand. Dev.	22,790,000,000	29.29	50.0%	11.75	2.32	15.75	3.05	8.46%	2,490,280,000	
1651 S&P1500 firms	Average	8,132,957,875	33.76	43.5%	17.39	6.53	23.31	1.57	8.55%	436,343,433	5.37%
	Median	1,372,900,000	26.30	38.1%	13.80	7.00	20.33	1.17	7.62%	50,275,714	3.66%
	Stand. Dev.	30,116,500,000	30.87	22.4%	12.60	2.68	17.98	1.38	5.41%	2,316,200,000	
1336 other firms	Average	515,400,897	16.61	90.1%	8.63	6.80	10.24	2.04	13.84%	53,922,380	10.46%
	Median	100,973,750	10.00	75.5%	5.70	7.25	8.39	1.14	11.67%	7,775,661	7.70%
	Stand. Dev.	2,929,090,000	24.04	61.5%	8.37	2.49	7.78	4.28	10.41%	2,564,190,000	

The 1651 S&P 1500 firms entered the S&P 500, Mid-Cap 500 and/or Small-Cap 500 firms in 1999

The 1336 other firms did not enter the S&P 1500 and had dilutive shares reported on SEC Edgar in HTML format.

**Table A-2**

**Full Sample Statistics Across CRSP Value Deciles**

	Decile -- N (S&P/Other Samples)	Total Equity Value	Stock Price (\$)	Volatility	Period of			S/X	Dilution	Employee Option Values	Employee Options/ Equity Value
					Listing	Maturity					
Average	1 -- 118 (0/118)	16,183,477	3.72	134.8%	10.30	5.97	1.71	16.44%	3,443,003	21.27%	
Median		9,342,190	2.55	123.1%	7.05	6.36	0.95	14.11%	1,129,800	12.09%	
Stand. Dev.		20,377,616	3.20	74.2%	8.06	2.22	4.86	10.44%	8,146,568		
Average	2 -- 135 (0/135)	41,407,816	5.62	120.1%	8.72	6.80	1.62	15.21%	7,032,296	16.98%	
Median		20,866,560	4.00	99.2%	6.50	6.96	0.94	12.67%	2,320,220	11.12%	
Stand. Dev.		58,278,792	5.63	72.8%	6.86	5.48	3.46	8.57%	11,808,104		
Average	3 -- 144 (1/143)	56,226,631	7.69	114.2%	9.65	6.57	1.66	14.65%	9,593,766	17.06%	
Median		38,668,250	5.88	90.7%	6.85	7.07	1.10	13.56%	3,532,132	9.13%	
Stand. Dev.		57,975,489	6.75	79.9%	8.60	1.94	1.67	9.07%	15,173,432		
Average	4 -- 170 (2/168)	109,580,202	13.08	97.6%	8.28	6.74	1.83	15.78%	22,292,534	20.34%	
Median		65,124,790	8.35	79.5%	5.65	7.22	1.27	12.94%	7,017,909	10.78%	
Stand. Dev.		166,658,373	15.22	65.3%	7.96	1.85	1.89	11.38%	47,441,391		
Average	5 -- 212 (31/181)	142,835,063	12.98	76.6%	9.64*	6.67	1.60*	12.96%	23,065,995	16.15%	
Median		90,117,875	9.38	65.5%	6.60	7.07	1.06	10.94%	7,131,832	7.91%	
Stand. Dev.		188,847,773	13.31	45.3%	8.54	1.76	1.85	9.04%	54,475,254		
Average	6 -- 238 (66/172)	266,902,173	16.75	69.3%*	10.41*	6.83	1.91*	13.3%*	40,677,834	15.24%	
Median		142,050,000	11.55	63.2%*	7.10	7.10	1.12	11.59%	10,263,586	7.23%	
Stand. Dev.		602,086,200	17.49	35.9%	9.73	1.64	3.30	10.40%	126,937,139		
Average	7 -- 313 (139/174)	398,144,746	21.23	63.4%*	10.78*	6.89*	1.82*	11.3%*	42,571,579	10.69%	
Median		261,550,000	16.40	55.7%	6.60	7.20	1.08	9.54%	14,752,876	5.64%	
Stand. Dev.		520,673,524	23.19	36.5%	10.13	1.67	3.36	9.34%	84,641,876		
Average	8 -- 406 (283/123)	850,699,508	26.67	53.8%*	12.81*	6.91*	1.80*	9.94%	80,536,537	9.47%	
Median		507,340,000	19.58	49.0%	8.25	7.18	1.16	8.89%	26,923,958	5.31%	
Stand. Dev.		1,141,490,415	23.10	27.0%	10.65	1.72	2.51	5.22%	178,518,869		
Average	9 -- 560 (482/78)	1,900,045,579	33.11	46.9%*	13.55*	7.02	1.83	9.2%*	172,559,959	9.08%	
Median		1,064,000,000	25.10	40.0%	8.20	7.05	1.18	7.90%	40,773,059	3.83%	
Stand. Dev.		3,330,277,337	30.10	28.7%	11.80	3.14	3.53	7.47%	528,394,931		
Average	10 -- 643 (610/33)	18,827,672,421	45.64	36.9%	22.05*	6.98	1.80	7.3%*	951,547,877	5.05%	
Median		5,893,700,000	36.90	31.1%	27.10	7.00	1.29	6.47%	171,260,000	2.91%	
Stand. Dev.		45,915,997,712	38.54	18.0%	13.09	1.33	3.01	4.93%	3,610,864,225		

\* Significant Average Differences

Sub-Sample/Deciles		5	6	7	8	9	10
Volatility	S&P 1500		52.43%	50.36%	49.83%	44.49%	
	Other		75.75%	73.74%	62.94%	61.50%	
Period of Listing	S&P 1500	14.60	16.64	15.96	15.05	14.51	
	Other	8.79	8.02	6.64	7.65	7.61	
Maturity	S&P 1500			6.66	6.76		6.94
	Other			7.07	7.24		7.77
S/X	S&P 1500	1.12	1.23	1.21	1.60		
	Other	1.69	2.17	2.31	2.26		
Dilution	S&P 1500		11.05%	9.55%		8.72%	7.27%
	Other		14.18%	12.26%		11.92%	8.81%

Employee Option Values - No significant differences

**Table A-3-a: Dilution across Industries**

SIC	Industry Name	S&P 1500 Firms			Other Firms			Mean with unequal $\sigma$	
		N	Average	Std. Dev.	N	Average	Std. Dev.	T	Prob
10	METAL MINING	9	4.78%	2.26%	3	3.99%	1.35%	-0.72	0.50
12	COAL MINING	1	3.42%		1	4.52%			
13	OIL AND GAS EXTRACTION	48	6.99%	3.79%	21	8.52%	4.62%	1.33	0.19
14	MINING AND QUARRYING OF NONMETALLIC MINERALS	3	5.14%	1.01%	2	2.58%	0.92%	-2.93	<b>0.08</b>
15	BUILDING CONSTRUCTION-GENERAL CONTRACTORS	11	9.74%	2.96%	7	10.55%	6.27%	0.32	0.76
16	HEAVY CONSTRUCTION OTHER THAN BUILDINGS	7	6.08%	3.36%	3	14.81%	3.33%	3.79	<b>0.02</b>
20	FOOD AND KINDRED PRODUCTS	43	8.29%	6.90%	25	11.64%	6.77%	1.95	<b>0.06</b>
22	TEXTILE MILL PRODUCTS	13	8.06%	3.15%	6	9.55%	3.98%	0.81	0.44
23	APPAREL AND OTHER FINISHED PRODUCTS (FROM FABRICS )	15	9.13%	4.64%	9	15.00%	7.66%	2.08	<b>0.06</b>
24	LUMBER AND WOOD PRODUCTS, EXCEPT FURNITURE	5	5.14%	1.44%	5	6.83%	4.75%	0.76	0.48
25	FURNITURE AND FIXTURES	10	5.32%	2.97%	8	12.37%	11.48%	1.69	0.13
26	PAPER AND ALLIED PRODUCTS	27	5.89%	2.39%	4	7.39%	4.04%	0.72	0.52
27	PRINTING, PUBLISHING, AND ALLIED INDUSTRIES	41	7.46%	3.84%	9	14.26%	7.59%	2.61	<b>0.03</b>
28	CHEMICALS AND ALLIED PRODUCTS	102	8.15%	3.86%	96	14.10%	10.69%	5.15	<b>&lt;.0001</b>
29	PETROLEUM REFINING AND RELATED INDUSTRIES	19	4.98%	2.64%	3	15.63%	10.50%	1.75	0.22
30	RUBBER AND MISCELLANEOUS PLASTICS PRODUCTS	14	7.73%	4.78%	15	13.66%	4.62%	3.40	<b>0.00</b>
31	LEATHER AND LEATHER PRODUCTS	5	8.13%	1.08%	6	9.17%	2.98%	0.79	0.46
32	STONE, CLAY, GLASS, AND CONCRETE PRODUCTS	15	6.36%	2.96%	8	14.42%	8.72%	2.54	<b>0.04</b>
33	PRIMARY METAL INDUSTRIES	30	7.73%	4.44%	10	8.98%	4.63%	0.74	0.47
34	FABRICATED METAL PRODUCTS	21	6.67%	3.07%	19	8.78%	6.31%	1.32	0.20
35	INDUSTRIAL/ COMMERCIAL MACHINERY AND COMPUTERS	118	9.71%	4.43%	84	14.20%	10.05%	3.83	<b>0.00</b>
36	ELECTRONIC AND OTHER ELECTRICAL EQUIPMENT	122	10.39%	5.13%	126	15.87%	8.51%	6.17	<b>&lt;.0001</b>
37	TRANSPORTATION EQUIPMENT	43	7.25%	3.66%	19	8.80%	3.92%	1.46	0.15
38	MEASURING, ANALYZING AND CONTROLLING INSTRUMENTS	53	10.13%	3.73%	89	14.10%	11.25%	3.05	<b>0.00</b>
39	MISCELLANEOUS MANUFACTURING INDUSTRIES	12	8.19%	3.80%	16	11.08%	9.42%	1.11	0.28
40	RAILROAD TRANSPORTATION	6	5.50%	1.91%	3	11.46%	2.88%	3.24	<b>0.05</b>
41	LOCAL AND SUBURBAN TRANSIT AND INTERURBAN TRANSPORTATION	1	19.67%		2	4.15%	4.93%		
42	MOTOR FREIGHT TRANSPORTATION AND WAREHOUSING	13	8.99%	4.17%	9	6.61%	4.23%	-1.31	0.21
44	WATER TRANSPORTATION	3	4.45%	3.01%	1	8.16%			
45	TRANSPORTATION BY AIR	19	9.94%	5.14%	4	12.89%	6.79%	0.82	0.46
47	TRANSPORTATION SERVICES	2	8.40%	2.47%	7	20.40%	20.35%	1.52	0.17
48	COMMUNICATIONS	59	8.52%	8.15%	25	15.62%	10.41%	3.04	<b>0.00</b>
49	ELECTRIC, GAS, AND SANITARY SERVICES	66	4.31%	5.73%	13	7.91%	3.70%	2.89	<b>0.01</b>
50	WHOLESALE TRADE&die;DURABLE GOODS	40	8.36%	4.87%	42	14.69%	10.99%	3.40	<b>0.00</b>
51	WHOLESALE TRADE&die;NONDURABLE GOODS	35	9.57%	4.58%	32	13.65%	9.35%	2.23	<b>0.03</b>
52	BUILDING MATERIALS, HARDWARE, GARDEN SUPPLY	2	5.21%	3.30%	4	11.70%	5.96%	1.72	0.17
53	GENERAL MERCHANDISE STORES	24	6.41%	3.54%	9	8.84%	2.49%	2.21	<b>0.04</b>
54	FOOD STORES	8	6.14%	4.35%	2	5.90%	3.41%	-0.08	0.94
55	AUTOMOTIVE DEALERS AND GASOLINE SERVICE STATIONS	4	6.94%	2.88%	3	10.49%	8.90%	0.67	0.57
56	APPAREL AND ACCESSORY STORES	19	7.95%	2.90%	16	12.05%	8.78%	1.79	<b>0.09</b>
57	HOME FURNITURE, FURNISHINGS, AND EQUIPMENT STORES	7	7.93%	1.76%	6	11.04%	5.08%	1.43	0.20
58	EATING AND DRINKING PLACES	26	10.95%	4.24%	24	17.93%	18.35%	1.82	<b>0.08</b>
59	MISCELLANEOUS RETAIL	26	11.34%	5.81%	23	10.91%	5.81%	-0.26	0.80
60	DEPOSITORY INSTITUTIONS	69	5.62%	3.26%	25	9.12%	4.77%	3.40	<b>0.00</b>
61	NONDEPOSITORY CREDIT INSTITUTIONS	21	7.43%	5.31%	22	11.43%	7.57%	2.01	<b>0.05</b>
62	SECURITY AND COMMODITY BROKERS, DEALERS, EXCHANGES	19	10.33%	5.73%	14	11.91%	9.43%	0.55	0.59
63	INSURANCE CARRIERS	60	6.65%	4.32%	33	9.40%	7.71%	1.89	<b>0.06</b>
64	INSURANCE AGENTS, BROKERS, AND SERVICE	3	13.83%	3.73%	3	6.94%	4.84%	-1.95	0.13
65	REAL ESTATE	2	6.50%	0.16%	11	11.61%	8.53%	1.99	<b>0.08</b>
67	HOLDING AND OTHER INVESTMENT OFFICES	53	6.19%	3.78%	59	6.71%	3.61%	0.75	0.45
70	HOTELS, ROOMING HOUSES, CAMPS	12	8.53%	4.22%	8	11.42%	2.58%	1.90	<b>0.07</b>
72	PERSONAL SERVICES	10	9.86%	5.93%	5	9.96%	2.91%	0.05	0.96
73	BUSINESS SERVICES	151	13.10%	7.97%	201	19.93%	14.26%	5.71	<b>&lt;.0001</b>
75	AUTOMOTIVE REPAIR, SERVICES, AND PARKING	5	8.84%	6.32%	4	10.17%	2.19%	0.44	0.68
78	MOTION PICTURES	4	9.90%	6.57%	10	16.27%	12.69%	1.23	0.25
79	AMUSEMENT AND RECREATION SERVICES	9	9.73%	2.60%	18	10.54%	6.65%	0.45	0.65
80	HEALTH SERVICES	24	10.68%	4.62%	33	14.99%	8.14%	2.53	<b>0.01</b>
82	EDUCATIONAL SERVICES	4	10.75%	7.09%	7	15.79%	9.11%	1.02	0.34
87	ENGINEERING, ACCOUNTING, RESEARCH, MANAGEMENT	18	10.53%	3.33%	33	17.55%	7.29%	4.71	<b>&lt;.0001</b>

**Table A-3-b: Volatility Estimates across Industries**

SIC	Industry Name	S&P 1500 Firms			Other Firms			Mean with unequal $\sigma$	
		N	Average	Std. Dev.	N	Average	Std. Dev.	T	Prob
10	METAL MINING	9	63.93%	34.34%	3	83.04%	45.10%	0.67	0.55
12	COAL MINING	1	53.69%		1	70.21%			
13	OIL AND GAS EXTRACTION	48	61.45%	18.27%	21	67.90%	48.15%	0.60	0.56
14	MINING AND QUARRYING OF NONMETALLIC MINERALS	3	58.09%	18.71%	2	153.06%	138.57%	0.96	0.51
15	BUILDING CONSTRUCTION-GENERAL CONTRACTORS	11	41.58%	12.64%	7	87.06%	79.15%	1.51	0.18
16	HEAVY CONSTRUCTION OTHER THAN BUILDINGS	7	48.71%	21.39%	3	82.15%	22.84%	2.16	0.10
20	FOOD AND KINDRED PRODUCTS	43	36.29%	13.35%	25	64.75%	49.40%	2.82	<b>0.01</b>
22	TEXTILE MILL PRODUCTS	13	59.09%	34.63%	6	86.92%	87.12%	0.76	0.48
23	APPAREL AND OTHER FINISHED PRODUCTS (FROM FABRICS )	15	57.64%	35.73%	9	94.65%	62.97%	1.61	0.13
24	LUMBER AND WOOD PRODUCTS, EXCEPT FURNITURE	5	51.35%	22.18%	5	54.03%	19.70%	0.20	0.84
25	FURNITURE AND FIXTURES	10	42.58%	9.46%	8	42.92%	20.86%	0.04	0.97
26	PAPER AND ALLIED PRODUCTS	27	37.98%	10.06%	4	48.94%	18.08%	1.19	0.31
27	PRINTING, PUBLISHING, AND ALLIED INDUSTRIES	41	36.21%	15.86%	9	67.93%	42.10%	2.23	<b>0.05</b>
28	CHEMICALS AND ALLIED PRODUCTS	102	57.40%	52.04%	96	105.83%	65.96%	5.71	<b>&lt;.0001</b>
29	PETROLEUM REFINING AND RELATED INDUSTRIES	19	39.79%	15.27%	3	136.61%	103.29%	1.62	0.25
30	RUBBER AND MISCELLANEOUS PLASTICS PRODUCTS	14	41.34%	16.23%	15	63.01%	32.43%	2.30	<b>0.03</b>
31	LEATHER AND LEATHER PRODUCTS	5	52.97%	6.88%	6	65.25%	23.04%	1.24	0.26
32	STONE, CLAY, GLASS, AND CONCRETE PRODUCTS	15	37.67%	18.06%	8	79.01%	37.82%	2.92	<b>0.02</b>
33	PRIMARY METAL INDUSTRIES	30	53.02%	25.27%	10	57.63%	19.34%	0.60	0.55
34	FABRICATED METAL PRODUCTS	21	38.38%	10.96%	19	67.80%	40.57%	3.06	<b>0.01</b>
35	INDUSTRIAL/ COMMERCIAL MACHINERY AND COMPUTERS	118	56.92%	28.21%	84	105.12%	74.79%	5.63	<b>&lt;.0001</b>
36	ELECTRONIC AND OTHER ELECTRICAL EQUIPMENT	122	67.69%	28.29%	126	108.23%	59.45%	6.89	<b>&lt;.0001</b>
37	TRANSPORTATION EQUIPMENT	43	44.79%	25.90%	19	71.99%	62.33%	1.83	<b>0.08</b>
38	MEASURING, ANALYZING AND CONTROLLING INSTRUMENTS	53	50.95%	21.13%	89	81.80%	38.63%	6.15	<b>&lt;.0001</b>
39	MISCELLANEOUS MANUFACTURING INDUSTRIES	12	49.79%	20.78%	16	77.18%	63.15%	1.62	0.12
40	RAILROAD TRANSPORTATION	6	38.90%	11.88%	3	55.76%	46.21%	0.62	0.59
41	LOCAL AND SUBURBAN TRANSIT AND INTERURBAN TRANSPOR	1	75.00%		2	146.99%	139.74%		
42	MOTOR FREIGHT TRANSPORTATION AND WAREHOUSING	13	52.85%	13.57%	9	69.84%	60.05%	0.83	0.43
44	WATER TRANSPORTATION	3	47.76%	22.00%	1	63.33%			
45	TRANSPORTATION BY AIR	19	46.77%	15.96%	4	81.37%	59.82%	1.15	0.33
47	TRANSPORTATION SERVICES	2	46.96%	10.07%	7	68.15%	26.43%	1.73	0.14
48	COMMUNICATIONS	59	58.84%	22.63%	25	80.16%	40.12%	2.49	<b>0.02</b>
49	ELECTRIC, GAS, AND SANITARY SERVICES	66	26.89%	13.15%	13	67.08%	26.72%	5.30	<b>0.00</b>
50	WHOLESALE TRADE&die;DURABLE GOODS	40	53.42%	24.86%	42	93.46%	43.85%	5.12	<b>&lt;.0001</b>
51	WHOLESALE TRADE&die;NONDURABLE GOODS	35	53.88%	20.83%	32	97.79%	67.14%	3.55	<b>0.00</b>
52	BUILDING MATERIALS, HARDWARE, GARDEN SUPPLY	2	75.51%	67.64%	4	73.33%	31.67%	-0.04	0.97
53	GENERAL MERCHANDISE STORES	24	46.74%	13.80%	9	70.19%	19.63%	3.29	<b>0.01</b>
54	FOOD STORES	8	45.13%	15.86%	2	201.92%	184.87%	1.20	0.44
55	AUTOMOTIVE DEALERS AND GASOLINE SERVICE STATIONS	4	59.18%	16.49%	3	87.72%	54.19%	0.88	0.46
56	APPAREL AND ACCESSORY STORES	19	55.93%	15.55%	16	81.44%	29.30%	3.13	<b>0.00</b>
57	HOME FURNITURE, FURNISHINGS, AND EQUIPMENT STORES	7	63.79%	14.98%	6	85.30%	33.92%	1.44	0.20
58	EATING AND DRINKING PLACES	26	46.22%	15.27%	24	77.08%	47.95%	3.02	<b>0.01</b>
59	MISCELLANEOUS RETAIL	26	68.99%	30.23%	23	87.39%	31.74%	2.07	<b>0.04</b>
60	DEPOSITORY INSTITUTIONS	69	32.51%	8.96%	25	46.13%	31.30%	2.14	<b>0.04</b>
61	NONDEPOSITORY CREDIT INSTITUTIONS	21	50.95%	34.97%	22	84.62%	70.16%	2.00	<b>0.05</b>
62	SECURITY AND COMMODITY BROKERS, DEALERS, EXCHANGES	19	48.93%	26.02%	14	57.61%	29.14%	0.88	0.38
63	INSURANCE CARRIERS	60	45.76%	26.53%	33	62.44%	58.94%	1.54	0.13
64	INSURANCE AGENTS, BROKERS, AND SERVICE	3	28.80%	6.65%	3	79.07%	45.29%	1.90	0.19
65	REAL ESTATE	2	41.82%	3.96%	11	42.43%	19.03%	0.10	0.93
67	HOLDING AND OTHER INVESTMENT OFFICES	53	36.83%	15.38%	59	38.72%	34.95%	0.38	0.71
70	HOTELS, ROOMING HOUSES, CAMPS	12	39.85%	7.81%	8	74.57%	45.00%	2.16	<b>0.07</b>
72	PERSONAL SERVICES	10	53.13%	24.13%	5	94.87%	64.30%	1.40	0.22
73	BUSINESS SERVICES	151	71.94%	30.33%	201	121.50%	75.77%	8.42	<b>&lt;.0001</b>
75	AUTOMOTIVE REPAIR, SERVICES, AND PARKING	5	64.27%	40.68%	4	85.70%	54.20%	0.66	0.54
78	MOTION PICTURES	4	48.96%	14.32%	10	96.46%	57.61%	2.43	<b>0.03</b>
79	AMUSEMENT AND RECREATION SERVICES	9	59.04%	24.54%	18	81.54%	53.05%	1.51	0.14
80	HEALTH SERVICES	24	56.11%	17.70%	33	97.42%	60.16%	3.73	<b>0.00</b>
82	EDUCATIONAL SERVICES	4	74.30%	19.85%	7	86.93%	21.11%	0.99	0.36
87	ENGINEERING, ACCOUNTING, RESEARCH, MANAGEMENT	18	69.38%	39.38%	33	105.50%	43.52%	3.01	<b>0.00</b>

**Table A-4: Employee Option Model Input Correlations**

	Exercise Proceeds*	Volatility	Maturity	S/X	Dilution	Granted Options	Period of Listing	Interest Rate	Dividend Yield	Employee Option Values
Total Equity Value	0.839	-0.082	-0.004	0.181	0.801	-0.068	0.106	-0.050	-0.016	0.790
Exercise Proceeds*	1.000	-0.085	0.001	0.082	0.928	0.039	0.098	-0.060	-0.011	0.796
Volatility		1.000	0.113	0.317	-0.018	0.345	-0.398	0.078	-0.232	0.032
Maturity			1.000	0.064	0.007	0.058	-0.132	0.069	0.008	0.007
S/X				1.000	0.146	0.150	-0.244	0.001	-0.122	0.246
Dilution					1.000	0.086	-0.269	0.061	-0.162	0.087
Granted Options						1.000	0.023	-0.048	-0.035	0.893
Period of Listing							1.000	-0.054	0.216	-0.026
Interest Rate								1.000	0.018	-0.035
Dividend									1.000	-0.045
Employee Option Values										1.000